Is it paradoxical to not be greater than God?*

Uwe Scheffler (scheffleru@philosophie.hu-berlin.de)
Institute of Philosophy, Humboldt University, Germany

Abstract. In his formulation of the ontological argument St. Anselm uses the relation of 'greater than'. Chr. Viger offers an argument to the effect that Anselm's proof succumbs to Russell's paradox, since the set of things which are not greater than God is paradoxical. Against this attempt I argue that the range of the relation depends on the domain and is therefore non-paradoxical.

Keywords: domain, nonexistent entities, onotological argument, relation, Russell's paradox

1. Introduction

In his *St. Anselm's ontological argument succumbs to Russell's paradox*[1] Christopher Viger shows how one can find a Russell type paradox in Anselm's proof. Clearly, Anselm would have wished to comment on Viger's argument. The following lines can be understood as an attempt to say what Anselm could have answered, if only he had known as much as Viger about set theory and modern logic and if he had seen Viger's essay. Such an answer, of course, does not provide an argument for the correctness of Anselm's proof. If the answer is convincing, it just shows that if Anselm is wrong, he is not wrong there, where Viger suspects him to be.

No philosopher really needs a motivation for dealing with the ontological argument. Nevertheless, Viger's new approach is connected with a critical observation: The most influential criticism of Anselm's argument, that of Kant, does not tell us which fallacy is underlying Anselm's reasoning (Viger, 123). This is not quite true, as Kant tells us[2] that existence is not a "real predicate", which means a predicate that contributes to the subject's notion and enlarges it. Using $\mathcal{E}$ for "existence" one can formulate Kant's criticism in a more formal way: Anselm forgot that for all properties f holds:

$$\forall y([\lambda x f(x)](y) \equiv [\lambda x(f(x) \wedge \mathcal{E}(x))](y)) \qquad (1)$$

The idea of having a property and the idea of being something and having that property are the same.

* I would like to thank Fabian Neuhaus and Yaroslav Shramko for helpful discussions.

I join in Viger's motivation in both respects: As Viger, I believe that Kant is essentially right. Moreover, since one can doubt Kant's conception of existence, formulated in the formula above, it would be highly appreciated to have an independent criticism. Hence, I feel the need to discuss Viger's important essay.

Section 2 contains a description of Viger's proof, as it is given in his essay, pp. 124f, and some observations about the universe of discourse. Section 3 is devoted to the problem of nonexistent entities. Together, they form an argument to the conclusion that Russell's paradoxical set is not one of the things God may be greater than.

2. Viger's argument

Viger's proof is presented here in a set theoretical language with the following abbreviations (due to set theory, Viger or introduced for convenience):

g – God
$>$ – greater than
Ω – things that God is greater than
$\mathbf{U}$ – Ω together with g
$\mathbb{R}$ – everything in $\mathbf{U}$ that does not contain itself
$\mathbb{D}$ – the universe of discourse, the quantification domain
– usual set theoretical and logical operators

2.1. Viger step by step

In the following outline of Viger's proof, quotations are of Viger's.

1. $\Omega = \{x : g > x\}$

   "Ω be the set of things that God is greater than" — by definition

2. $\mathbf{U} = \Omega \cup \{g\}$

   "$\mathbf{U}$ be the set consisting of everything in Ω together with God" — by definition

3. $\mathbb{R} = \{x : x \in \mathbf{U} \wedge x \notin x\}$

   "$\mathbb{R}$, which contains anything in $\mathbf{U}$ that does not contain itself" — by definition

4. $\mathbb{R} = g \vee \mathbb{R} \neq g$

   "Either $\mathbb{R}$ is God or $\mathbb{R}$ is not God." — Law of the excluded middle

5. If $\mathbb{R} = g$, then $\mathbb{R} \in \mathbf{U}$

   — by definition of $\mathbf{U}$

6. If $\mathbb{R} \neq g$, then by $\forall x(x \neq g \supset x \in \Omega)$: $\mathbb{R} \in \Omega$ and therefore $\mathbb{R} \in \mathbf{U}$

   — by definition of Ω and $\Omega \subset \mathbf{U}$

7. $\{x : x \notin x\}$ is contradictory (Russell)

   — if $\mathbb{R} \in \mathbb{R}$, then by definition of $\mathbb{R}$ holds: $\mathbb{R} \notin \mathbb{R}$; if $\mathbb{R} \notin \mathbb{R}$, then it is not the case that $\mathbb{R} \notin \mathbb{R}$, that is: $\mathbb{R} \in \mathbb{R}$

8. $\mathbb{R} = \{x : x \in \mathbf{U} \wedge x \notin x\}$ is contradictory, hence not defining a set

   — paradoxical conditions do not define sets

9. $\mathbf{U}$ is not defining a set

   "since $\mathbb{R}$ ..., a subset of $\mathbf{U}$, the contradiction tells us that $\mathbf{U}$ is not a set" — $\mathbb{R}$ is not a set (is contradictory), hence from $\mathbb{R} \subset \mathbf{U}$ follows, that $\mathbf{U}$ is't a set neither

10. Since g is not problematic, $\mathbf{U}/\{g\} = \Omega$ is problematic, that is: not defining a set.

    — since God is a (singleton) set, $\mathbf{U}$ would be a set if Ω were one by definition of $\cup$

11. Since $\{x : g > x\}$ is not defining a set, $>$ is not a correctly introduced relation.

    "the defining property of Ω must be self-contradictory"

2.2. Ω AND $\mathbf{U}$

Viger describes Anselm's 'greater than' relation as one that "must obtain between God and all things that are not God" (Viger, 124). Since a binary relation is usually modelled as a subset of the Cartesian Product of the domain with itself, it is quite natural to speak about sets: We are looking for all elements of the domain to which g stands in relation $>$. But what does "all things" mean? Let $\mathbb{D}$ be the universe of discourse, than "all things" can mean only "all elements of $\mathbb{D}$". The property of being not greater than God is than a set, the set of just those elements of $\mathbb{D}$ which are not greater than God. One is not allowed to change $\mathbb{D}$ *en passant*, including new or excluding some elements from the domain of quantification. Therefore, one has to decide the question whether sets or relations (that is: sets of n-tuples) are *elements* of $\mathbb{D}$ or not.

Suppose they are. Then, the 'greater than' relation is defined for sets and relations, and – by supposition – God is greater than any set in $\mathbb{D}$. It is not interesting how God manages to be greater than a particular man, a certain event, the number 9 and the set of letters in the English alphabet. Viger reminds us that in natural languages we usually understand greatness in some particular respect: physically, mentally, more important or whatever else. From an abstract point of view, greatness – seen as a set of pairs of elements of $\mathbb{D}$, – turns out to be an union of all these particular relations (possibly adjusted to transitivity and antisymmetry). But, and this is important, in order to be a domain, $\mathbb{D}$ has to be a set. If sets and relations are members of $\mathbb{D}$, one has to ensure that $\mathbb{D}$ is still a set and nonempty. In particular, $\mathbb{D}$ must not contain $\mathbb{R}$ or its relatives.[3] Since $\mathbb{R} \notin \mathbb{D}$, it is neither the case that $g > \mathbb{R}$, nor $g \not> \mathbb{R}$. $\mathbb{R}$ is, considered from this perspective, simply nothing which could be meant by "everything".

So, if $\mathbb{D}$ is a set and contains sets and relations as elements, one has to check how $\mathbb{R}$ turned up at **U**, which is $\mathbb{D}$ (presumably, this is Viger's intention) or a subset of $\mathbb{D}$. This is the topic of the second part of section 3.

Let $\mathbb{D}$ be a universe which does not contain sets and relations *as elements*. That does not mean that one cannot refer to properties or relations, that "being greater than" is not expressible. One would do it by using predicate constants instead of individual constants (names). Again, $g > \mathbb{R}$ and $g \not> \mathbb{R}$ aren't even expressible, because $\mathbb{R}$, being a set, is expressible only as a predicate and cannot enter a relational sentence at an argument place.

Summarizing the discussion of the first two lines of the reconstruction of Viger's proof we find: If $\mathbb{D}$ is a properly constructed set, it is not the case that $\mathbb{R} \in \mathbb{D}$. Since $\{x : g > x\}$ is constructed as a subset of $\mathbb{D}$ it cannot contain $\mathbb{R}$ as an element. Viger, as far as he is concerned, apparently tries to construct the universe of discourse (his **U**) out of a property, built from a relation and a name, and a singleton set. What Anselm could insist on is that there is no such relation prior to the universe.

What is really wrong with $\mathbb{R}$?

3. Nonexistent entities

3.1. $\mathbb{R}$

Viger's definition of $\mathbb{R}$ is given in section 2.1 in step 3; and at the very end of the paragraph where it is introduced, Viger comes to the conclusion that $\mathbb{R}$ is paradoxical (Viger, 125):

> If $\mathbb{R}$ contains itself, then by definition it must be a member of **U** that does not contain itself. If $\mathbb{R}$ does not contain itself, than it is something in **U** that does not contain itself and so is in $\mathbb{R}$ by definition.

These conclusions are expressed in lines 5 and 6. It is quite important to see that Viger – as he in fact tried to do several lines above the quotation – has to guarantee that $\mathbb{R} \in \mathbf{U}$. For, provided the definition is correct, $\mathbb{R}$ is not a member, but at most a subset of **U** and the second horn of the dilemma in the quoted conclusion can not be derived. Everything depends on the idea that $\mathbb{R}$ is *something* which is exceeded by God. If $\mathbb{R}$ is *nothing* in contrast to *the empty set*, it cannot be element (or even subset) of a universe. What's the difference?

Viger convinces the reader that – in a set description via a propositional function as $\{x \in A : P(x)\}$ – P can be a nonsense or contradictory predicate, since in that case the set in question is simply empty. But there is a crucial difference between a set with a contradictory propositional function and a contradictory set:

$$\sim\exists x \quad x \in \{x : P(x) \wedge \sim P(x)\} \tag{2}$$

$$\mathbb{R} \in \mathbb{R} \wedge \mathbb{R} \notin \mathbb{R} \tag{3}$$

The first sentence describes an empty set: No entity is member of the set. The set itself is subset of any set and, in an appropriate language, may be an element of other sets. The second sentence, describing the behavior of $\mathbb{R}$, claims that some entity is and is not element of a set and that some set contains and does not contain an entity. From (3) one can imply (in classical logic) any other sentence. Since one can derive it from the definition of $\mathbb{R}$ if this is not restricted, $\mathbb{R}$ is not a set and, moreover, because it is defined as a set it is not an entity at all. $\mathbb{R}$ is not in the universe and Anselm could blame his critic for incautiously comparing God to *unthinkable* entities.[4] This, of course, was not intended by the ontological argument.

A short inspection shows that a definition of $\mathbb{R}$ as in step 3 is completely innocuous in many cases. Consider a universe $\mathbb{D}$ which contains no sets at all and the set of entities which do not contain themselves as elements: This set is even not empty, but the universal set. Set theorists are quite clever in creating harmless environments.

But, nevertheless, there is Viger's application of the law of the excluded middle, and the reader is faced with the possibility that **U** may contain a contradictory element. Can one, after all, deal with nonexisting entities?

3.2. Is $\mathbb{R}$ God?

No, it isn't. The law of the excluded middle, as in

$$\mathbb{R} = g \quad \vee \quad \mathbb{R} \neq g \tag{4}$$

is correctly used in Viger's proof. Since $\mathbb{R}$ is not God (who is "unproblematic"), the second part of the disjunction must be discussed. A natural language sentence as "$\mathbb{R}$ is not God" has two interpretations with respect to the logical properties of *not*: First, *not* means the standard negation of propositional logic, semantically claiming that "It is not true that $\mathbb{R}$ is God". The law of the excluded middle is stated for such an interpretation and $\mathbb{R} \neq g$ might be (and is in fact) true due to unfulfilled existence presuppositions – $\mathbb{R}$ does not exist, hence the second disjunct of (4) is true. In a second, different but related interpretation "$\neq$" is understood as a relation itself, which holds or does not hold between the entities of $\mathbb{D}$. It seems sensible to differentiate between those interpretations on a syntactic level, too, for example by using sentential and predicate negation:

$$\mathbb{R} \neq g \text{ is either } {\sim}(\mathbb{R} = g) \text{ or } \mathbb{R} \overset{\neg}{=} g \tag{5}$$

As we have seen, the truth of ${\sim}(\mathbb{R} = g)$ is not dependent on the existence of $\mathbb{R}$ (or even g), while $\mathbb{R} \overset{\neg}{=} g$ is a predicative sentence. Semantically, $\mathbb{R} \overset{\neg}{=} g$ is interpreted as true if and only if the pair consisting of the interpretations of $\mathbb{R}$ and g is element of the interpretation of $\overset{\neg}{=}$ – that is, $\mathbb{R}$ has an interpretation which is element of the universe. Sentence (4) with $\mathbb{R} \overset{\neg}{=} g$ instead of $\mathbb{R} \neq g$ is not a logical truth: Both $\mathbb{R} = g$ and $\mathbb{R} \overset{\neg}{=} g$ can be false because $\mathbb{R}$ is not existing ($\neg$ is contrary but not contradictory).[5] Correspondingly, Anselm would arraign Viger's steps 4 and 6 by claiming that 4 is not logically true because of the properties of $\neg$, or that 6 is defect because $\forall x(x \neq g \supset x \in \Omega)$ does not hold: There are entities x (in particular, $\mathbb{R}$), for wich ${\sim}(x = g)$ holds but not $x \in \Omega$.

4. Conclusion

Russell's paradox entered the ontological argument because Viger interpreted Anselm as *creating* the universe of discourse, the interpretational domain of quantifiers like "all things", "everything", by fixing the range of the relation "greater than" with respect to the argument "God". He does not challenge the matter what the domain of the relation "greater than" is and presupposes it to be paradoxical. Relations are defined as subsets of sets of (corresponding) n-tuples of elements of a base domain, so the paradoxical character of Viger's implicit domain must be inherited by all other considered 'sets'. Instead, Anselm used (the set of) all existing and thinkable things in order to define the property "to be exceeded by God", which does not force a Russell-paradoxical construction.

Notes

[1] Christopher Viger: "St. Anselm'sontological argument succumbs to Russell's paradox". *International Journal for Philosophy of Religion* 52 (2002), pp. 123–128

[2] Immanuel Kant: *Kritik der reinen Vernunft.* Verlag Philipp Reclam jun., Leipzig 1979, 655

[3] Somehow, it sounds even odd to say: $\mathbb{R}$ cannot be in $\mathbb{D}$ (and $\mathbf{U}$) Since, there is literally *nothing about* which we can say such things.

[4] "Medieval philosophers recognized that God could not understand what is logically impossible, ..." Viger, 126

[5] There are several attempts to characterize a predicate, or inner, or strong, or predicative negation $\neg$, a philosophical discussion and a complete formal theory is found, for example, in Horst Wessel: *Logik* (Logos Verlag, Berlin, 1998). Aside from unfulfilled existential presuppositions, there might be other reasons for considering a case of $\sim f(i) \wedge \sim\neg f(i)$. Hegel called sentences as *The spirit is sweet (or green, or square) or the spirit is not sweet (or not green, or not square)* trivial and not worth pronouncing them (Georg Wilhelm Friedrich Hegel: *Wissenschaft der Logik II.* Akademie-Verlag, Berlin 1975, 57).

Address for Offprints:
Institute of Philosophy
Humboldt University
Unter den Linden 6
10099 Berlin
Germany