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INTRODUCTION

Time plays a fundamental role in everyday life as well as in most sciences and historical disciplines. Science as a systematic study into the natural, social and human spheres of life examines all sorts of processes that take place in time. But, apparently, science has very little to say about the way we think about time and the nature of time itself. This is because we cannot define time in terms of any other concept; on the contrary, we use it in the specification of other concepts. Time seems to be a fundamental concept which we have to accept as a precondition for our understanding of our own life and the whole universe around us. This does not imply, however, that time itself cannot be an object of inquiry, or that it can only be grasped intuitively. Even if we cannot provide a formal definition of time, something instructive and important about time can always be said concerning how it is related to other fundamental concepts like space, event, thing, causation, free will and human experience.

We experience that things, as well as our thoughts, come into being, change or move around in space, and eventually come to an end as parts of a flowing process of events which are not present to the mind all at once. More than anything else this experience is what makes us ascribe temporal features to the world. The overriding question is, therefore, whether these changes or transformations of things should be regarded as forming the basis of our experience of an objective time flux, or whether the consideration of such changes as a manifestation of becoming is merely a projection of a subjective element onto them. Being experimentally based themselves, empirical sciences seem unable to give a final answer of how far the experience of such changes can be given an objective basis. Thus, it has always been an important issue of philosophy to answer (i) what is time really? (ii) how do we acquire objective knowledge about the nature of time? and (iii) how is, can, or should time be represented by our thinking? Which of the possible representations of time does correspond with real time, how can we justify such a claim of representation, and does it make sense to distinguish between different representations of one single and universal time? Indeed, all these questions are intimately related. On the one hand, it is impossible to determine how we get to know objective time without the slightest idea of what time is. Similarly, it is impossible to decide that a representation illustrating our thoughts about time is objective without knowing what time really is. On the other hand, whether real time might be essentially different from the way we actually experience time or not is an open question.

In a programmatic way it could be said that the physics of time deals mainly with what we say about the nature of time from an empirical perspective, whereas the philosophy of time deals mainly with what time really is from a metaphysical perspective, and the logic of time deals with possible ways of reasoning about time from a semantic perspective. Indeed, the physics of time seeks to provide an account of time's ontology. To say, however, that physics of time is relevant for our understanding of the nature of time is not entirely uncontroversial. For if time is something over and above physical processes, then the nature of time has to be different from its physical manifestation. So how can physics then say anything essential about time? And even if one considers physical processes as providing evidence for the nature of time, they may yet amount to no more than evidence in the sense that time cannot ontically be reduced to physical processes. Answering such questions lies outside the scope and the power of empirical sciences, it seems, and is left to the means and the skills of philosophy. Nevertheless, whatever the nature of time may be, physical processes are very often regarded as an indication of what time is, simply because these processes cannot possess features that are excluded by time's nature.

If the nature of time is considered to be something over and above its physical manifestation, it would certainly be denied that physical processes can be taken as conclusive evidence for the nature of time. Instead, it might be argued that our experience of a temporal becoming and of free will is fundamental for our understanding of time. Thus, we all have an immediate experience of the flux of events in the present, and we all see ourselves as having a real choice between possible alternative actions in the future but not in the past. So any representation leaving out these fundamental experiences of being a human being would, according to such a view, fail to give an objective account of every aspect of time.

Whether one believes that the discourse of time can or cannot be reduced to the discourse of physical processes, the belief has considerable consequences for which other beliefs are considered possible. Starting out from our temporal experiences and the way we talk about them, one would usually insist on a kind of representation of time which is quite different from that taken by a person who starts his analysis from the character of physical processes. Still both types of representation of time seem fundamental for our talking about time, and it finally becomes a philosophical question which of them are ontologically the most fundamental. The present collection of papers seeks not only to establish some of those possible representations and to entertain which of them are the most basic. It also deals with the different features time can have within these representations, depending on what is believed to warrant the ascription of objective features of time within a certain representation.

Since Aristotle three models of representation are established as the most common: the numerical, the spatial and the branching tree model. Thus, Aristotle saw time as the number of movement in respect of before and after. The nature of time is not movement but only the enumeration of movement (cf. [3], 219b). The practical correlation of this idea is the fact that we count time in seconds, minutes, hours, days and years, and we have watches and calendars to help us in doing so. The concept of time as numbers was taken over by philosophers in the renaissance when classical mechanics eventually replaced Aristotle's physics. Isaac Newton simply described time as a mathematical quantity which is not related to anything external. As he said "Absolute, true and mathematical time, of itself, and from its own nature, flows equally without relation to anything external, and by other name called duration" ([34], p. 6). Consequently, time is something over and above physical processes. Absolute time has to be distinguished from relative time: "relative, apparent and common time, is some sensible and external (whether accurate or inequable) measure of duration by means of motion, which is commonly used instead of true time, such as an hour, a day, a month, a year." After Newton's definition of absolute time physicists became used to see time as a numerical parameter in the laws of nature. And this idea gave rise to the thought that real numbers fully represent moments or temporal instances. Such real numbers are the values of an external time variable.

The second model of representation can be seen as a development of the first model. For according to this, time can be specified as a set of intrinsic number coordinates, and it is therefore natural to consider time as a dimension like the three dimensions of space. Even in everyday language we use spatial metaphors whenever we say that something covers or spreads over a certain amount of time. In physics time as dimension became a well-established concept with the introduction of Einstein's special theory of relativity. Although the spatial and the temporal interval were shown to vary relative to an observer, Minkowski proved that the union of space and time into a space-time would allow for the specification a four dimensional interval that would be invariant to any observer. Of course, by treating time as a dimension it is natural to think that time could consist of more than one dimension. Such an idea has been entertained by several people in the past. The latest shoot on the stem is Stephen Hawking's suggestion of a complex time of two dimensions formed by adding an imaginary dimension of time to the real one. Moreover, if time is described as a dimension analogous to the three dimensions of space, one might start wondering about the possibility of time travel in such a model similar to what is possible with regard to space travel. The conclusion, however, might very well be that one cannot travel in time for which reason time regarded as a fourth dimension has to be very different from the three dimensional space.

The third model of representation does not construct time as a single line but as a branching tree, and is commonly associated with the notion of an objective becoming. The basic idea is here that at every moment time consists of several possibilities for a future course of events. As time goes by these different possibilities are narrowed down to only one actual course of events. The model provides a nice way of representing our talk about counterfactuality in the sense that one can show what would have been the case if someone had made another choice in the past. Thus, branching time seems to take seriously the notion of a free will, since our choice happens to be between some real alternative possibilities. Moreover, such a representation has the advantage that by making a difference between the past and the future it depicts an asymmetry in time. The present and/or the past consist of ontologically determinate events, but the future contains ontologically indeterminate events. Even some interpretations of quantum mechanics introduce the idea of branching time in order to represent the possible outcomes of probability distributions.

Finally, a fourth model of representation should to be mentioned. Although it is no longer accepted in our culture, it played a fundamental role in antiquity and still plays a crucial role in other cultures. The idea of time as cyclic equates time with a circle. This model stems from the fact that many processes seem to repeat themselves over and over again: day and night succeed one another, and seasons follow each other year by year. Thus, it is not so far-fetched to imagine the resurrection of individual events and things. A modern variant of this idea can perhaps be found in Gödel's solutions to the field equations of the general theory of relativity that permit the existence of closed causal chains. Given the structure of space-time by his field equation solutions, it should be possible for any test particle, following a geodesic curve, to revisit every earlier and every later event, and *in principle* be possible for a rocket to make a round trip into any region of the past, the present and the future.

1. THE PHILOSOPHY OF TIME

The philosophical discussion of the nature of time and which properties time really possess starts with Aristotle. The succeeding debate between temporal substantivalism and temporal relationism seeks an answer to the first question, whereas the debate about whether tenses are real or not seeks an answer to the second one. In his *De Interpretatione* we find a first rough formulation of (or at least hints at) many of the problems discussed below.

1.1. *Aristotle: The future sea–battle*

It is hardly possible to overestimate the intellectual influence of Aristotle's text. In modern time it has given rise to considerations in tense logic, modal logic, and many-valued logic (cf. [2], [28], [55], [41], and others), as well as stimulating discussions about the ontological status of the future, about the intellectual soundness of fatalism, and about the relation between truth and time. To see why, we should try to give an interpretation of the final, concluding paragraph of Aristotle's famous Chapter 9 of [4]: that it is *not necessary* that of every affirmative sentence and its negation one is true and the other false. In order to do that we follow the line of Aristotle's argument.

(1) — An Introduction.

In the first paragraph Aristotle repeats the idea of Chapter 7 that every affirmation or negation about things, that were or are, has to be true or false. This is by no means surprising and is nothing more than the claim that statements are characterized by there ability to be true or false. Moreover, for many sentence pairs, one being an assertion and the other the corresponding negation, one has to be true and the other has to be false. This goes with "universals taken universally" ('Every man is white — not every man is white'), and particulars ('Socrates is white — Socrates is not white'). It doesn't work for "universals not taken universally" ('A man is white — a man is not white'), which can be true together, but not be both false. Nevertheless, in any case, and independently of what kind of negation we use, both sentences are true or false. For particulars that are going to be, the picture becomes different. In this case the argument is as follows.

(2) — From truth to necessary existence.
If every affirmation or negation is true or false, then for two such sentences uttered by different people, one of them utters the truth. Because the statement is true, matters cannot be otherwise than as it claims. Hence, what it claims is necessarily the case just as it is. A similar argument holds for negation, and so Aristotle sums up: If for every statement the affirmation or the negation is true, then what it claims is necessarily the case or necessarily not the case.

(3) — Future existence is necessary existence.
Suppose that something is the case *now*. Then every statement uttered earlier claiming that it will be the case was true. Moreover, it *always* was true, and because of that, it was always impossible for the thing which is the case not to happen. A similar argument holds for things which are not the case and negated sentences, so Aristotle draws the conclusion: Everything that will be will happen necessarily.

(4) — Meeting a possible argument.
One might try to avoid the necessary existence of the future by arguing the premiss of (2): both sentences are false. This leads to absurd consequences, because in that case, from "There will be a sea–battle tomorrow" and "There will not be a sea–battle tomorrow", it would follow that the sea–battle would neither happen nor not happen tomorrow.

(5) — From necessary existence to determinism.
Back to (3). If it were to be the case that all things happen necessarily, then there would be no door open to chance; people would be unable to prevent things from happening, and it would make no difference how people act.

(6) — From indeterminism to real possibilities.
The consequence of (5) is simply not true: people know of many things

which they can influence. The possibility of a thing's acquiring certain properties is open just because this is not actually realised yet. Not everything that happens, happens of necessity. Even though one of two contradictory sentences is true, the other might well have been.

(7) — From real possibilities to truth.
This paragraph makes essentially two claims:

1. From the fact that for all things it holds that if they exist, they exist necessarily, it does *not* follow that they exist necessarily.
2. From the fact that there necessarily exists one or another entity, it does *not* follow that the former necessarily exists, or the latter necessarily exists.

The failure of these entailments is due to the fact that a thing that comes to have a certain property does not always have the property, and an event which will come into being does not always exist. The sea–battle tomorrow for instance, is such an event: If it occurs, it occurs necessarily, but it is not necessary that it occurs; and it is necessary that at any time a sea–battle will occur or not will occur, but it is neither necessary that it occurs at this time nor necessary that it does not occur at this time. As statements are true according to how actual things are, Aristotle claims for things that are not but may possibly be or not be:

(8) — The famous conclusion.
It is not necessary that of every affirmation and corresponding negation one is true and the other false.

If Aristotle's argument is correctly reproduced here, its strength clearly depends mainly on (2), (3) and (5). Paragraph (4) simply meets a possible objection, and closes a branch of argumentation. Both (6) and (7) complete the indirect argument by *modus tollens* using the empirical fact that people are free to act in different ways. The argument as a whole forms an indirect proof. Of what?

Paragraph (2) presupposes that of every statement and its negation one is true and the other false ((4) rules out the possibility of both being false). This is the claim which is to be attacked, so Aristotle assumes it in order to refute it by deducing a contradiction. In (3) he seems to argue with a variant of the first rejected principle of (7): If from existence it follows that the entity necessarily exists, it exists necessarily. In a modal language with $\Box$ as a modal operator and $\mathcal{O}$ an appropriate predicate (existence, occurrence, or even truth), the principle is:
From $\forall x(\mathcal{O}(x) \supset \Box\mathcal{O}(x))$ it follows that $\forall x\Box\mathcal{O}(x)$.

Aristotle is absolutely right in the first part of paragraph (3) since he postulates the existence *now*, he is allowed to infer from $\forall x(\mathcal{O}(x) \supset \Box\mathcal{O}(x))$ and from the actual existence of a thing to its necessary existence. But what he proves is only "Everything that is, happened necessarily", and not "Everything that will be, happens necessarily". Since the argument is based on the latter sentence, there is a gap in the argumentation. Aristotle was clearly aware of this fact and uses it in (7). It is not our task to discuss the relation between a fixed future and the possibility of free actions here. Since his opinion is quite commonly held, we take it for granted that (5) is correct, and that a completely determined future excludes the freedom of action. Similarly, we accept the empirical existence of free actions as postulated by Aristotle. Then (6) is correct too, and we arrive finally at (7). Besides the principle of quantified modal logic mentioned above, one of the principles of propositional modal logic is found here. In a propositional modal language it is formulated as follows:
From $\Box(p \vee q)$ it does not follow that $\Box p \vee \Box q$.
Aristotle first gives an interpretation in terms of necessity and occurrence: If there is necessarily a sea battle or no sea battle, then it must not be the case that there is necessarily a sea battle or there is necessarily no sea battle. Since statements reflect how things are, he continues, the same should hold for the sentences. This allows for a second interpretation: If it is necessary for one of two sentences to be true, neither the one nor the other need to be necessarily true. They might *not yet* be true. For Aristotle, as for all of us today, $\Box p \vee \Box \sim p$ is obviously not a logical truth.

The formulas $\Box p \vee \Box \sim p$ and $p \supset \Box p$ cannot be accepted in a modal logic; with $\Box$ interpreted as "it will hold in the future that" or "it will hold at a certain moment in the future that" they lead to absurdities. On the contrary, the principle Aristotle accepted is, intuitively, clearly interpretable and entails the converse Barcan formula. The explicit reference to the temporal aspect of the argument means that logics with tense operators can capture this interpretation more satisfactorily than can other (alethic) modal logics and semantic conceptions with more than two truth values, or truth value gaps. The analysis of Aristotle's argument shows that the grammar of time is connected with that of truth and existence. His appeal to the freedom of action nicely demonstrates the interaction of logic, epistemology, metaphysics and ethics.

1.2. *Augustine: Before God created heaven and earth, he did nothing*

Augustine's most well-known contribution to the philosophy of time consists in a discouraging cry: What, then, is time? And he complains that so long as nobody puts the question to him, he knows the answer. But, if he is to explain it to somebody who does ask, he doesn't know the answer. In Book 11 of his *Confessiones* (cf. [5]) Augustine is concerned with three important metaphysical questions: What was *before* the beginning of time? What is the ontological status of tenses? How can we measure temporal distances?

Augustine converted from rather materialist and pantheist positions under the influence of Neoplatonic philosophy. In what followed he recognized a God, who alone exists truly, who is an unchangeable being, and who is ontologically superior to all changeable beings. Accordingly, since heaven and earth are changing, they are created. What is changing has to be created, and since God is not created, he is unchangeable. Time is change, in a sense, because it consists of a past pushed away by a future, a future born by a past, and a present related to both of them. So they are created. In the world of God, there is no temporal flux, what there is, is simultaneous and the entirety is present.

The question about the beginning of the time, then, is transformed into two theological questions: What did God do, before he created the world? Why (and when) did he decide to create the world just at the time he did?

Augustine first draws a parallel to the spatial aspect of the problem: *Where* could God have created the world? Obviously, he couldn't do it anywhere, because space is only created together with the world. In the same sense, God does not *act in time* before time. What could he have done? He could have created a being. But this would have belonged to the world, and even God is unable to create a being *before* he created a being. Thus, before he created heaven and earth he did nothing. God created time, for there could not be any time *at which* God could have decided to create it. Equally, there could not be any time flowing before time was created, so neither question can be answered in a material manner. Time was created together with the world, and there is no time outside the world. As Augustine claims very clearly: God is *not* prior in time to times, for otherwise he wouldn't be prior to any time.

We don't want to leave this topic without mentioning for the reader one of the few jokes to be found in the older philosophical literature. Augustine mentions a straightforward and conclusive answer to the question about what God did before he created the world: He created a hell specially for those people who want to know too much.

When it comes to tenses, Augustine argues in a very anti–realist manner: in order to be future, something has not to exist, but to come into existence, and in order to be past, something has not to exist, but to have existed. So, non–existence is plainly a presupposition of the future and the past: they cannot exist without loosing their properties of being future and past. Moreover, a present without a past and a future, which was not future and never will be past, and therefore which is eternally present, is not time, but eternity. How, then, can one say that the present exists if it depends on the non-existing past? One can't. Consequently, neither the past nor the present nor the future exist.

Augustine's next argument concerns duration. Sometimes one claims 'Something lasts a long time', say, a hundred years. What does it mean for a hundred years to persist? It is at most one year that is present and being, ninety nine either already no longer exist, or do not yet exist. Moreover, regarding the present year, only one day is present and exists; three hundred and sixty four are either already no longer, or do not yet exist. We have to define as present something in time that cannot be divided into even the very shortest moments — but this will be so small as to be without duration. So there is no time which may last for any duration: if it lasts, it is separable into past and future; the present moment has no duration.

Now, there seem to be empirical arguments in favour of the existence of tenses. As Augustine mentions, it has to be explained how the prophets saw the events they had foreseen. It is obvious to him that what does not exist cannot be foreseen. Equally, telling true stories about the past would be impossible if the storytellers can't see past entities in their mind. And here is the key to the solution: His childhood, Augustine claims, is past and therefore no longer exists, but the images and thoughts that he sees while speaking about his childhood are present, because they are in his memory. Correspondingly, if one thinks about future actions and calculates risks and advantages, the actions still do not exist yet. The thoughts and calculations are present, and when one starts to act, the action will become present, too. Future things do not exist, but their causes and marks exist. Whoever sees the future does not see future events themselves, but their present causes and marks.

In a strict sense, then, the tenses past, present, and future do not exist.

Instead, one has to say that there is a presence of the past, a presence of the present, and a presence of the future. These modi are given *in the mind*, as the present memory of the past, the present experience of the present, and the present anticipation of the future. Of course it is possible to speak about the non–existing past and future, and about the present without duration as if they were existents: there are three tenses. This is how we usually speak. But Augustine maintained that this manner of speaking is incorrect.

There is no past and there is no future, and the present has no duration. How can one know that one time interval is twice as long as the other? How can we measure it? Not in the future or in the past, because what does not exist cannot be measured. Not in the present, because that has no duration.

First Augustine discusses a conception according to which time is the movement of the sun, the moon, and the stars. Within the confines of such an approach different temporal intervals could simply be related to fixed movements of the sun. Take, for instance, a day, to which belongs a complete movement of the sun from its place on the eastern horizon and back to the same place (we would speak today of a complete rotation of the earth). What, Augustine asks, is a day — the movement itself, its duration, or both? If it is the movement, then one should speak of a day even in the case in which the sun suddenly completes a rotation in an hour. If it is the duration, then one should speak of a day in the case of twenty–four such movements in the situation mentioned. If both, then one couldn't speak of a day in either case. If the sun needs half of the time in order to complete a period, it is quite unclear what it then means to claim that this interval is half as long as the usual one. By the way, there is a supplementary argument against this approach in the Bible: when God was asked to halt the sun and did so; time continued to flow nevertheless.

Augustine answers in terms of his subjectivistic time theory. One doesn't measure events which do not yet or no longer exist, but something which is memorized in the mind. The impression of bygone things exists in actuality, so it can be measured in the present. This is true even in the case of the future: if somebody wants to cause a sound for a certain period, he has to "go through the period silently", put it into his memory, and start to generate it. This, according to Augustine, will last until the sound reaches the determined end. The present intention to perform the sound makes the future turn into the past, the future decreases and the past increases until there is no future and the whole is gone. All this is possible only because the human mind is able to perform a threefold task: to anticipate, to experience and to remember. This is the basis of time.

According to Augustine, time is not a property of the empirical world. The real being, God, is completely out of time anyway, God is eternally in an untemporal sense even. Time is not an independent property of events or things, and neither the past and the future, nor earlier and later, exist objectively. It is rather an outcome of the soul, or the mind that creates and connects past and future by the mind's abilities to expect and to anticipate, to remember and to reproduce. It may seem that this conception is close to Kant's one, but this is clearly nothing but on the surface.

1.3. *Kant: A pure form of sensual intuition*

Kant's transcendental aesthetics is based on an ontology that recognizes a persisting substance as the basis of changes in appearances. His answer to the general question "How can we know?" depends on his thesis that the content of perception *is structured*, and is not an unorganized chaos. This is due to two *a priori* facts: There are the categories, concepts which are constitutive to the world of appearances, and there are the forms of pure intuition, equally constitutive. The categories guarantee, for instance, the existence of a cause for each event; the forms of pure intuition are space and time. All appearances are given to us only in space and time.

For Kant, time is not an empirical concept which could be subtracted from experience (cf. [21], pp. 103 ff.). In order to gain an understanding of what time is, one has first to isolate the empirical intuition by disregarding all conceptual import contributed by the reason. Then, when all kinds of sensation have been ignored, the remainder must be *a priori* and a case of pure intuition. One of the pure forms of sensual intuition is time. If time were not *a priori* and part of our perceptual abilities, there wouldn't be simultaneity and temporal sequence in perceptions. It is impossible to experience appearances outside time, but it is possible to consider empty time, in which the appearances have been taken out. The main characteristics of time therefore follow from its *a priori* character, and not from experience:

1. Time has only one dimension.
2. Different times follow one another, and are not simultaneous.
3. Different times are only parts of one and the same time.
4. Time is infinite, *i. e.*, it is not bounded.
5. All parts of time are time.

This amounts to a topology of time which Kant postulates, and is modeled by a continuous, infinite, and actually given straight line. Its properties are as *a priori* as time is itself and prior to all possible experience. The three *modi* of time — persistence, sequence, and simultaneity — guarantee the unity of perception: for every change there is something persistent that changes, for every change there is a rule, and simultaneously existing phenomena are interrelated (cf. [21], pp. 272 ff.).

Time is unique, one–dimensional, and finite. But Kant thinks that the question of its reality arises and must be addressed. For on the one hand, time is merely a subjective condition of human perception. As such, time is nothing in the world of things in themselves, because these are not subjects of perception. In that sense it is false to claim 'All things are in time'. On the other hand, time is real in the realm of human perception; there can never be a subject of perception that is not in time. Therefore it is true to claim 'All things, as appearances, are in time'. Kant discusses a possible attack against the empirical reality, but transcendental unreality, of time: Since changes are real (at least the changes of our own ideas), time must be real because of the fact that there would be no change without time. He answers that if there were a kind of non–sensual perception, it would yield a knowledge to which neither time nor change occur anymore.

One of Kant's major contributions to the examination of time is his first antinomy of pure reason. We are interested here only in the temporal part of it (cf. [21], pp. 514 ff.):

- Thesis: The world has a beginning in time.
- Antithesis: The world has no beginning and is infinite in time.

It is well–known that Kant defends both the thesis and the antithesis. His argument is indirect:

- Thesis: Suppose the world has no beginning. Then, for every arbitrarily chosen time point, there is an infinite sequence of moments which preceede it. But the infinity of a sequence means precisely that it cannot be completed. Therefore, there is no completed infinite sequence before every time point. Therefore, a beginning in time exists.

- Antithesis: Suppose, the world has a beginning. A beginning is a moment before which there is some time but in which the thing still does not exist. With respect to the world as a whole, this must be empty time. But in an empty time the beginning of anything whatsoever is impossible, because in no part of this time does the condition for the beginning obtain. Therefore, a lot of things can begin in the world, but the world itself cannot begin. Therefore, no beginning in time exists.

Kant's solution to the antinomy is in line with his philosophical premisses: Both thesis and antithesis of the antinomy are false because the world is that of appearances. In reality, time does not exist. The idea of a complete totality belongs to the world of things in themselves, and to such a world the concept of time cannot be applied.

According to Kant, the world of appearances is partly a product of human mental activity. His subjectivistic strategy allowed him to solve the puzzle of the first antinomy essentially by rejecting it. The transcendental idealism leaves no room for time at all, and the empirical reality of time goes together with its *a priori* character. But then no doors are open for an empirical investigation of time and tenses. The mere possibility of a different temporal geometry would already be a blow against the whole idea. As in case of the *a priori* geometry of space (where Kant believed the Euclidian to be the only possible one), different concurring models have been found for time, too (cf. [35], pp. 87 ff.). These topologies offer other possibilities for the solution of the first antinomy of pure reason.

1.4. *McTaggart: The unreality of time*

Kant and Augustine motivate their subjectivism concerning time in different ways. For Kant, time is (transcendentally) unreal because it belongs to the realm of appearances; for Augustine, time is unreal because only the present can consistently exist. Both are participants in a long tradition that goes back as far as to the 5th century BC, when Parmenides, and later on Zeno of Elea, showed motion and change to be inconsistent, and therefore time to be unreal. A highlight within this tradition, and a starting point of the contemporary discussion of the reality of time, is McTaggart's proof of the unreality of time (cf. [31]).

He believes that nothing that exists can be temporal, and so time has to be unreal. His argument proceeds in two stages. First he shows that earlier–later relations of all kinds are not understandable as *temporal* without the past–present–future distinction. Then, in the famous part of his argument, McTaggart shows this latter distinction to entail inconsistencies.

McTaggart begins with the observation that all temporal differences are differences of either the one or the other kind: either an event can be earlier or later than another event (simply earlier, or by a certain number of seconds, minutes, etc.), or two events differ in being past, present, or future (simply past, or one deeper in the past than the other, etc.). He says that the former difference creates a *B series*, while the latter generates an *A series*. All observations of events in time are observations in an A series or in a B series. The point is now that events in an A series differ essentially from events in a B series in one important respect: events *never* change their position in a B series, while they are *always* changing their position in an A series. So, if Caesar crossed the Rubicon and *later on* came to power, then his crossing the Rubicon ever was and ever will be the earlier event of both. By contrast, the position in the A series changes: first Caesar's crossing the Rubicon was in the remote future, then in the near future; it became present, and thereafter part of the close past; now it is in the remote past and becomes ever more remote.

In order to prove that the B series cannot constitute time independently, McTaggart supposes that time involves change. The question turns on whether changes are possible in a B series alone, without incorporating an A series. But this is impossible. Suppose an event be earlier than another; then, because the events are in a B series, they always were and ever will stand in that relation. One cannot say that one of the events becomes an event, or ceases to be one. Similarly, one cannot say that one event changes into another — because that would presuppose the end of an event and the beginning of another one. All properties an event can have outside an A series are permanent: if Caesar's crossing the Rubicon was fast, it was and always will be fast; and if it was important to history, it was so and will remain so. There is only one property that seems to offer any hope of grasping real change, and that it is the position of an event in the A series. According to McTaggart, then, there is no change without the A series, and thus no time without the A series.

To prove the unreality of time, then, it is enough to prove the inconsistency of the A series. If the A series is inconsistent, it cannot exist, and it cannot constitute time. Moreover, nothing else could constitute it, and so time must be unreal. In order to show the inconsistency of the A series McTaggart considers the following sentences:

1. A past event cannot be present or future.
   A present event cannot be past or future.
   A future event cannot be past or present.
2. Every past event has been present and future.
   Every present event will be past and has been future.
   Every future event will be present and past.

The first sequence of sentences simply tells us that A series characteristics are incompatible, they are mutually exclusive. The second sequence of sentences mirrors the fact that if the A series were real, every event must have all A series characteristics. Both facts cannot be had together, at least if the grammatical tenses of the sentences in the latter sequence are ignored. Lets try to eliminate them:

3. Every past event is a past present event and is a past future event.
   Every present event is a future past event and is a past future event.
   Every future event is a future present event and is a future past event.

But here we run into serious trouble: by definition, an event has to occupy all temporal positions in the A series, and some of them are incompatible. In order to avoid this we have to re-introduce grammatical tenses by reformulating 3.:

4. Every past event has been a past present event and has been a past future event.
   Every present event is a future past event and is a past future event.
   Every future event will be a future present event and will be a future past event.

We are thus able to eliminate the grammatical tenses. But even if we are able to keep the contradiction at bay on any given level of this procedure, it will always arise again on the next level. So the reality of the A series leads to contradiction, and must therefore be rejected.

McTaggart's proof still generates a lot of discussion, especially of its second part. The proof is usually explicated with the help of tense operators mapping untensed sentences into their tensed variants 'It is now the case that ...', 'It has been the case that ...', and 'It will be the case that ...'. Then our temporal intuitions suggest a modal semantics in which McTaggart's contradictions are really derivable. This has convinced many people of the correctness of this part of McTaggart's proof, and for them, at least the unreality of tenses is proven. The unreality of time would then follow from this argument if the first part of McTaggart's proof were valid too. A position rejecting the first part of McTaggart's proof but accepting the second, and so combining the unreality of tenses with the reality of time, is possible (cf. for instance [12], pp. 137 ff., and [32]). The first horn of the dilemma is already dulled by the reflection that things, rather than events, are what change. According to this conception events in a B series are really unchanging and persistent: Caesar's crossing the Rubicon still has all properties it had and will have them permanently. What may change are Caesar and the Rubicon; they do have different properties at different times. But if there are real changes, there is real time — despite the fact that there are no real tenses.

McTaggart's proof is objectionable in many other ways. The first part depends both on the idea that change is not definable in a B series, and (formally) on the idea that B series positions remain unchanged. But clearly change is definable simply by the *difference* between events of different B series positions. When elaborated, this definition need not involve any A series terms. Regarding the second presupposition, B series positions might conceivably be thought to change in other ways than by changing position in the B series ordering. Perhaps at first it is not true that Caesar's crossing the Rubicon is earlier than his coming to power because there are no events of crossing and coming to power. Then, when these events have come to be, the B series statement is true, and remains so until such time as mankind has completely forgotten about Caesar and his doings, when the B statement is no longer true.

The objections against the first part of the proof are independent of the acceptance of the second horn of the dilemma. A possible objection to this consists in introducing not only means for the expression of tenses (that is, usually, tense operators), but also the possibility of explicitly expressing truth (preferably a truth predicate, or even a truth operator on sentences). As has been shown, the expressive power of such a language allows for an explication of the idea that all events must occupy all A series positions in a way that is free from contradiction (cf. [25]). In that case, of course, there is no longer any contradiction and the paradox dissolves.

1.5. *Tensed sentences and tenseless truth conditions*

All theories so far provide an answer — or at least a partial one — to the question of how entities exist in time. It has, however, always been of special interest, what these theories say about *the future*. First, we can ask whether there is an ontic difference between the future and the past. If the answer is yes, we should handle them in an *asymmetric* manner, mirroring the discrepancy. If it is no, we must be able to explain all the main differences we feel in everyday life. Finally, we must admit that the abilities of remembering and influencing have at least a predominant direction. We remember the past, but not the future, and we affect the future, but not the past. Second, we can treat past and future analogously and ask whether they differ from the present. If the answer is no, then we have to reject all tenses and affirm our commitment to the reality of time; if it is yes, then we would probably favour a kind of temporal solipsism, recognizing the existence of the present alone.

Aristotle, for instance, would have promoted a conception drawing a line between the past and present at one hand, and the future (or most of it) at the other. He believed that it is not yet true whether or not there will be a sea battle tomorrow, and so it is for many other sentences about the future. As for sentences about the present and the past, they do have a definite truth value. The differences between human capacities with respect to the past and the future are build in: we remember the past because it is definite, we act towards the future because it is still indefinite.

Augustine addresses all tenses alike. Because memory, experience, and expectation are all functions of the mind, there is no essential ontic difference between the tenses. A difference could only come out by considering the epistemic dependency: memory is memory of former experience, anticipation is anticipation of later experience. But this is, of course, a difference between the various functions of mind. And our cognitive abilities concerning the past and the future are different because they are constituted by different mental functions. Nevertheless, real time does not exist according to Augustine.

For Kant, time is a necessary condition of perception, and that is a function of man, of his perceptual apparatus, not of nature. Kant's topology of time suggests no difference between past, present, and future. The problems arising from this understanding of time are addressed, for instance, in his famous antinomy about determinism and free will. We shall return to that problem below.

McTaggart, we saw, systematically addressed the fact that temporal references are given by two different techniques, using the idea of an A series and a B series to disprove not only the reality of tenses, but also of time. Now, he crucial point in one of the objections to McTaggart's attempt to reduce B series events to A series events consists in the observation that events in the B series are *fixed*. Since this is so, we can consider facts (the entities what make true sentences true) to be *tenseless*: If an event occurs at t, then it does so seen from every time point, be it earlier, simultaneous, or later than t. But then, as will become clear in the next paragraph, we should be able to formulate tenseless truth conditions for tensed sentences — an idea, which was first expressed by Andersen and Faye in [2], but independently developed by Mellor in [32]. The basic insight is discussed there and in [11] and [12].

In order to clarify the idea, consider a series of sentences:

(1) Caesar crosses the Rubicon now.
(2) Caesar has crossed the Rubicon.
(3) Caesar will cross the Rubicon.
(4) Caesar crosses the Rubicon at t.
(5) It is true that Caesar crossed the Rubicon at t.
(6) It is true at some t' that Caesar crossed the Rubicon at t.
(7) It is true for all t' that Caesar crossed the Rubicon at t.

Let the sentences be tokens, in the sense of singular utterances at a certain place and time. (1) to (3) are tensed sentences whose truth values vary with the time of utterance: Before Caesar's crossing the Rubicon, (1) and (2) are false, and (3) is true; respectively in the case of the utterance of these sentences during or after his crossing the river. A realist account of tenses would explain this by the existence of a tensed fact: there was a future event, moving through the present to the past and step by step changing the truth value of (1) to (3). Those convinced of the unreality of future tense would have to admit that — in the case of (1), for instance — a truth value is first altogether lacking, then becomes true, and finally false. What happens if we look at (4)? There is, of course, no change in the temporal location of Caesar's crossing the Rubicon at t since (4) is a tenseless sentence. According to the position recapitulated here, the existence of truth conditions of a sentence does not depend on our abilities to *know* them. So, the position according to which (4) has no truth value before t and becomes forever true or false from t on, depending on whether the crossing took place at t or not, is explicitly repudiated. To be completely clear, (5) alone, which expresses the existence of a tenseless fact, is enough for ascribing a truth value to (4). If such an account goes together with the idea that sentences ascribing truth values are *tenseless*, then (5), (6), and (7) are *logically equivalent*. What, then, do we have so far?

First of all, provided that Caesar crossed the Rubicon at t, (1) is true at t if and only if (4). The truth condition for (1) is given in a tenseless sentence. Notice that (1) and (4) are not equivalent! Because of the equivalence of (5), (6) and (7) (the corresponding tenseless fact exists eternally and forever), any of them can be used to ascribe truth conditions to (2) and (3), which are true at t' if and only if (4), and t' is later or earlier, respectively, than t. Again, we do not have to refer to A series terms at all — they have tenseless truth conditions. The outcome is obvious: *just because we have real time, that is, real positions in a B series, we can well be skeptical about tenses and the A series*. Tenses are dependent on B series terms because they have tenseless truth conditions.

So, if (4) provides us with tenseless truth conditions for (1), we might try to get rid of tensed sentences all together. This is possible in many cases and contexts, where tenses are frequently eliminated, along with other indexicals. But this is impossible in the case of beliefs and actions. Anyone who believes (1) has a true belief if he has it at t. But this still does not mean that he believes (4). It is possible to have a false belief in (1) a day before t. Conversely, it is not possible to believe in (4) and have a true belief in (1) simply by thinking at t that the time is a day later. Actions based on (1) or (4) may therefore turn out to be completely different. It is easy to construct such examples concerning knowledge too, and it is by no means surprising in view of the discussion above that the subjective activity of the mind plays a fundamental role there.

The argument rules out the first part of McTaggart's proof of the unreality of time by showing that the A series is not prior to the B series — rather the opposite is true. Together with the second part of McTaggart's paradox, we are left in a situation where two ways are open: Either one can draw no ontic distinction between the future, the present, and the past, and reject the reality of tenses (time exists as B series positions only, equally real), or one can attack McTaggart and (or) the premisses of the above investigation. One of these premisses has been disputed by objecting to the claim that truth conditions are tenseless (cf. [38]). They are sentences about the truth of sentences, and at least these might be considered as tensed: Suppose somebody says (3) a day before t. Then the sentence "(3) is true", taken as a sentence token, is tensed — it is true the day before t, and it is false two days later. Technically, this step would lead to tensed predicates of truth, or at least to an urgent need for further investigations into the area, where tense logic, the logic of existence and theory of truth overlap.

1.6. *Fatalism and free will*

The problem we are going to deal with now is not only a theoretical one; it is anchored deeply in our common sense beliefs governing human behavior.

On the one hand, we believe that things change by rules, that similar initial conditions allow similar causes to produce similar effects, that the laws of nature do not allow for exceptions, and that human motives, wishes and desires, fear and hope can and often do determine behavior as well as physical forces. This fact is basic for our conception of nature, but it is also essential for our ideas about punishment, justice, and education, and it is fundamental for our conviction that we are able to change things by ourselves, and be responsible for what we do. It is this thought of a *rule* that leads to the notion of determinism and which can be formulated in various ways. The main idea is that how the world is and what happens in it depend on other states of the world and other occurrences in the world. Usually, this dependence is thought to involve a strong temporal direction: effects are dependent on their causes, and they follow them — a topic which is addressed below. But if the causes are already *given*, then the effects are inevitable. The latter are, though not part of our experience, as real and unavoidable as their causes. An affirmative sentence concerning the occurrence of a situation is true or false at every moment because it is part of the closed past or present, or because the causes of future events' occurrence or absence already exist at this moment.

On the other hand, people know by introspection that they are able to act in accordance with decisions made by themselves. People maintain, they have had (in some situation) several different possibilities for acting, and they say that they could have done otherwise. This feeling is so strong that Aristotle used it, as we saw, without further argumentation in order to refute bivalence for sentences about the future. Since Hume, we know that two problems are involved here: The first is the freedom of action — if one performs an action or refrains from doing so, and if there are no physical forces necessitating this performing or refraining (in virtue of bonds, a paralysis, or maybe a fatal menace), and if one were to have chosen otherwise, one could not have performed the action. The second is the problem of the freedom of the will — are we free to want what we want? In a sense, the first problem is dependent on the second. Excluding wishes, aims, and intentions from the general realm of determinism, for instance by rejecting any causal influence of the physical on psychical states, but recognizing them as free causes starting new causal chains, allows for freedom of the will and *therefore* for freedom of action. But is it really possible to decide to do otherwise than what was done? What role does temporal reasoning play in such arguments?

Jan Łukasiewicz defines 'determinism' without reference to causality, but *via* truth and time (cf. [28], pp. 113 ff.). According to him, determinism is characterized by the belief that if a sentence A is true at a temporal instant t, it is true at any instant earlier than t, too. In order to describe this conception he uses the beautiful picture of a film, in which all of us participate. At the moment, we are in the middle of the film, and no one knows the end of it. But it is there, the ending exists from the beginning of the performance, because the picture is completed. Consequently, all true future propositions are already true now, and there is no real difference between the future and the past: it is as impossible to influence the future in accordance with one's wishes, aims, and fears, as it is to influence and to change the past. Determinism has only one recommendation of how to behave: watch the movie, and patiently await its end. As was mentioned, it is *bivalence* that is the main target of Łukasiewicz, and his feeling of uneasiness about these consequences was one of the major reasons for developing the idea of many-valued logics. However, he sees another possibility for avoiding determinism while accepting a certain understanding of general causation. Łukasiewicz accepts a principle of causation which is based on the following notion of a cause: a fact F at t is cause of a fact G at s if t is earlier than s and it is possible to infer the statement describing G from known laws and the statement describing F. The principle of causation, then, is the proposition that every G at s has a cause F at t (earlier than s), and that at every moment between t and s there is a fact which is both the effect of F and the cause of G. According to Łukasiewicz, every cause of an effect has a cause itself, and the relation of 'being a cause' is transitive. Usually, these properties (transitivity and generality of the causal relation) are considered to be sufficient to prove infinite regress, and therefore determinism. Łukasiewicz shows why this is not necessarily so. He provides us with a model in which both transitivity and generality hold, but determinism (in his sense) fails. Imagine the facts to be modelled by points on an interval $\langle 0, 1 \rangle$ in accordance with their temporal location. Let the present moment be somewhere in the first half of the interval, and a certain future fact be at the end of the second half of the interval. Then, *all* infinitely many causes of the fact in question can be located in the second half of the interval, coming closer and closer to point '$0,5$', but never reaching it. At the present moment, there is no cause of this fact, however, and determinism fails. Nevertheless, due to the continuity of time, that fact has a cause, which itself has a cause, and so *ad infinitum*. It is not difficult to transform this argument into another dealing with 'being an effect' instead of 'being a cause'.

Łukasiewicz proved the possibility of keeping (a part of) the future open, while accepting a general principle of causation in quite a strong form. Unfortunately, this is not an argument in favour of free will or free actions. One can debate such a solution simply by claiming that an action, or a decision to act in a certain way, precisely means to *set up* a cause. We act (or decide to act) in order to obtain effects, otherwise this wouldn't be a conscious deed: action requires intentionality. Then, once performed, it has infinitely many causes because it is a cause. One couldn't have done otherwise — especially if one did it intentionally. This is one of the main background ideas made explicit in the following argument against free actions.

Peter van Inwagen's notion of determinism is much alike that of Łukasiewicz's, but deals with the world as a whole. As he puts it ([51], p. 65), 'determinism' is the conjunction of two ideas:

1. for every moment there is a proposition expressing the state of the world at this moment;
2. if A and B are propositions that express states of the world at some moments, B follows from A and the laws of nature.

Van Inwagen's argument (discussed in detail in [50], and quoted here without reference to the concrete content of the example) results in the claim that *it is impossible to think of a performed action as not being performed*. It runs as follows:

Let B be a proposition expressing the state of the world at t including some action of an agent a performed at this time, and A a proposition that expresses the state of the world at some t' much earlier than t (perhaps before the birth of a). Let L be the conjunction of all laws of nature. Suppose a could have done otherwise at t. Then, according to van Inwagen, we have an indirect argument:

(1) If determinism is true, then $A \wedge L \vdash B$.
(2) If a had done otherwise at t, B would be false.
(3) If (2) is true, a could have made B false.
(4) Then, a could have made $A \wedge L$ false.
(5) Then, a could have made L false.
(6) Agent a couldn't have made L false.
(7) Hence, if determinism is true, a could not have done otherwise at t.

The problem with this agument clearly rests on two independent assumptions: one about truth, and one about existence in time (and backward causation). The second assumption is hidden in how to obtain (5): if one is able to make a conjunction false, and one of the conjuncts deals exclusively with facts occurring before one's birth, one must be able to make the other conjunct false. That's why the argument is not circular — if t' is simultaneous with, or later than t, there would be a clear case of begging the question. Past facts are hard facts, so to speak; they exist and do not change any more. For van Inwagen, it is even more obvious that sentences about past facts cannot be made false than that laws of nature cannot made false. From these hard facts follows, in his view, the logical necessity of all performed actions. But there is a further difficulty, too, concerning determinism. If determinism is true, propositions describing the state of the world for every moment of time follow from the laws of nature and a particular description of the state of the world at a certain time. This concept of determinism requires that there are sentences describing every future occurrence of events, and *that they are true now* because one can derive them from sentences which are true now. Various objections can be made against such a position as van Inwagen's.

First of all, not everybody would accept the view that sentences describing the (complete) future state of the world are true or false now. Further, even if they were, it might for various reasons be doubted that one can know their truth value. Secondly, it is not obvious that even if one knows a sentence about the future to be true, this sentence is known to be *necessarily* true. And last but not least, the underlying concepts of law of nature and causation are also objectionable. Every one of these possibilities has been used in order to escape fatalism: There is still no future, hence it is open. The future exists, but sentences about the future become true or false in the process of realization of the corresponding events. Being involved in the genesis of the future by acting is a principal obstacle to our knowing that part of the future, so *our* future is generally open to *us*. A future event can be causally determined in the sense of the existence of events causing its occurrence in the future, but even in that case it need not necessarily be logically determined in the sense of being a truth–maker for a corresponding sentence. Backward causation may be possible, or causation may not be transitive. These escape routes from fatalism lead into the semantical and physical theories of time.

2. THE PHYSICS OF TIME

Is time something over and above physical processes? And if it is, is time then a substance or an entity which can subsist independently of physical events? This issue has dominated the debate on the physics of time for centuries. Ever since Newton and Leibniz, two opposing views of time (and space) have been promoted. In modern jargon, one position is called substantivalism, and the other is named relationism. Newton argued that (1) time is a substratum that underlies physical events and processes, and relations among such events and processes depend on temporal relations intrinsic to the substratum of time. Leibniz argued that (2) time is nothing other than the relations among physical events and processes; time is not a substratum in which the events and their relations get their properties from some underlying temporal instances and relations. In his general discussion Newton treated time and space on equal terms as coordinate time and coordinate space because his main argument for substantivalism was connected with absolute motion in the form of rotation. For to take place absolute motion requires both absolute time and absolute space. Nevertheless among Newton's contemporaries there were arguments that dealt with absolute time in particular.

2.1. *Absolute* versus *Relational Time*

Already Newton's teacher Isaac Barrow thought that time exists independently of motion, and he saw time as a precondition of physical existence. Barrow developed an argument which he believed proved that time existed regardless of physical events and processes ([6], p. 5-6). A similar argument was later used by Samuel Clarke in his correspondence with Leibniz when Clarke acted as a go-between between Newton and Leibniz ([1], p. 49). The argument runs as follows:

(i) God created the world at a time t.
(ii) God could have created the world at a time t^* earlier than t.
(iii) If it is possible for God to create the world at t^*, then the time instance t^* must exist.
(iv) Hence, time must exist before t.

The argument is valid as it stands. The only way to challenge it would be to raise doubts about one of the premisses. The most obvious one would be (iii). This is exactly what Leibniz did in his reply to Clarke. We may represent Clarke's understanding of (iii) in modal form as $\Box(\Diamond p \supset q)$, whereas Leibniz could only accept a reading of (iii) as $\Box\Diamond(p \supset q)$. What Clarke claimed in terms of truth was the statement: 'It is true in every world W that if p is true in a possible world W^*, then q is true'. In other words, for every world W it is the case that if God created a different world W^* at t^*, then t^* also exists in W. Leibniz, however, argued that God's capacity had to be understood differently. The modal interpretation he gave of (iii) was 'It is true in every world W that if p is true in a possible world W^*, then q is true in *this* particular world'. Thus, with respect to any world W it is the case that if God created a world W^* at t^*, then t^* exists only in W^*. God may have the power to create the actual world when he liked but there would be no time before this creation took place. His conclusion was therefore that Clarke's argument was not valid.

Around the same time John Locke came up with another argument for absolute time ([23], bk. II, ch. xiv, §30). Assume, he said, that (i) some event X happened before the creation of the world Y, and (ii) nothing happened between X and Y. Then (iii) the interval between X and Y would still have taken up a certain amount of time. This is so because of the true counterfactual 'If the sun, or some other periodically moving thing, had existed when X happened, then it would have completed a certain number of revolutions between then and the time when Y happened'. Locke's argument suffers, however, from the same weakness as that of Barrow and Clarke. Unless one can show by the evaluation of the above counterfactual that the material conditions of the antecedents can be fulfilled in our world, then one could, like Leibniz, say that what is the case in one possible world need not be the case in the actual world. It seems that the existence of substantival time must be proved in some other way and not by appeal to a certain interpretation of modal expressions because such an invocation of possible worlds already presupposes either a substantival or a relational view of time in possible worlds.

Before Kant reached his so-called critical period, he believed that space and time were ontologically prior to bodies and events existing quite independently of our mind. In his 1768 essay *Concerning the Ultimate Foundation of the Differentiation of Regions in Space* he introduced a new argument for the reality of absolute space that didn't rely on the Newtonian idea of absolute motion. Imagine, he said, that the first created thing was a human hand. It would necessarily be either a right or a left hand. The relationist claims that such a body should be described in the terms of relations among the parts of the hand. But such a description would be exactly similar for both right and left hands. Therefore, Kant concludes — or so it seems — that it is only with respect to a real absolute space that the internal difference of incongruent counterparts are distinguishable. But if the relations among the underlying spatial points of the hand are not different from the relations among the material points of the body, however, how can this argument then prove the reality of substantival space? John Earman (cf. [10], ch. 7) has nicely shown that neither the substantivalists nor the relationists can hope to benefit from Kant's argument.

Be that as it may, the question here is: can we imagine an argument for absolute time analogous to the one Kant intended about absolute space? Kant never came up with one, and nobody else seems to have suggested one. Nevertheless, there seems to be one which Hume had already indirectly proposed (although he himself would probably have done everything to resist it). We have never experienced that an event causes another which does not temporally succeed it. Imagine some causal connection obtains between two events. How can we order these causally connected events so that the description of them will provide us with a causally asymmetric relation? According to Hume, there is no other way in which we can determine which of them is the cause and which is the effect than by referring to their temporal order: the cause necessarily occurs earlier than its effect. Obviously, what we have here is an argument for absolute time, because it is the intrinsic order of time itself which yields the asymmetric causal relation.

2.2. *The Causal Theory of Time*

The causal theory of time is an attempt to meet such an argument by producing a relational theory of time. It is a theory which does not only seek to establish that various attributes of time are ontologically based on causal properties, but its proponents, like Hans Reichenbach and Adolf Grünbaum, also hope to be able to reduce instants of time to physical happenings. A result of such an analysis would, of course, be that no moment can exist unless a physical event takes place. Yet any relational theory of time, and so the causal theory, must face the problem of explaining how actual temporal instants and relations are related to possible events and causal relations. An even bigger problem for the causal theory of time is how to define temporal relations in terms of causal relations. Success requires distinguishing causes from effects independently of any reference to an intrinsic direction of time.

Thus time's arrow is perhaps the most serious issue for any relational theory of time. From our usual conception of time in terms of past, present and future we believe that time contains an intrinsic and inherent orientation. Assuming now that time can be reduced to physical processes, or at least physical processes can be used as direct evidence for the structure of time, it might be thought that physical laws would reveal those causal features that give time its direction. Such features, however, are difficult to distinguish. For the laws of classical mechanics are essentially time invariant. This means that the reversal of the time parameter t to $-t$ in Newton's equations of motion is equivalent to the reversal of the velocities v to $-v$ of the system under investigation. Moreover, this general feature of Newton's laws can also be found in the corresponding laws of motion in quantum mechanics, electrodynamics, and the theory of relativity. All laws of motion allow a reversible process to proceed in either direction. Among physicists this feature is then interpreted as an intrinsic time symmetry of these laws. Should one or other temporally directed process not be found in nature, that is merely because some boundary conditions are not practically realizable. The conclusion is that mechanical and electrodynamical laws cannot supply the world with time's arrow.

2.3. *The Thermodynamical Argument for Time's Arrow*

In order to find a physical basis of the direction of time, many physicists and philosophers of science have turned to other processes. In fact, only very simple macroscopic systems with few degrees of freedom fulfill the ideal picture of reversible processes of mechanics. Complex and compounded macroscopic systems having many degrees of freedom are often irreversible, and they are subject to the laws of thermodynamics. The first law can be stated in terms of the internal energy E, the heat Q and the work $W = p\,dV$ as

$$dE = dQ - dW.$$

Now, in the case of reversible processes the heat is restored along the path of a closed loop, $\oint \frac{dQ}{T} = 0$. Thus we may write the first law of thermodynamics in terms of the perfect differential quantity $\frac{dQ}{T} = dS$:

$$dS = \tfrac{dE}{T} + p\,dV,$$

where S is a new function called entropy. But in the case of irreversible processes the closed integral does not vanish, since it is impossible to return the system and the surroundings to their original state. Instead we must use the second law of thermodynamics.

According to the second law of classical, non-statistical thermodynamics, the entropy of a closed system S will always move from a lower state to a higher state untill the system reaches its state of equilibrium where the entropy has obtained its maximum value. Thus, we have that

$$\Delta S \geq \int \tfrac{dQ}{T},$$

where the equality applies only to strictly reversible processes. Consequently, the changes of the energy state of the system invariably take place in the direction of increasing entropy, and this direction cannot be reversed. Moreover, the increase of entropy obviously *coincides* with time's arrow, which suggests using entropy to define a temporal direction of the system.

Thus it seems possible to attach time's arrow to an objective law. The only problem was that thermal systems can be seen as systems build up by particles obeying mechanical laws. Ludwig Bolzmann was able to prove that the entropy in a closed system can be characterized by an a priori probability for the distribution of the motion of the molecules of the system. He also proved that every change of the entropy consists of a change in the distribution of the motion of the molecules in the direction of one with a greater a priori probability. In this way Boltzmann apparently succeeded in deriving the irreversibility from very simple assumptions about a reversible classical system and thereby solved the conflict between reversible and irreversible features of the system.

Boltzmann's result is based on his famous H function defined by

$$H = \int d^3v f(\boldsymbol{v}, t) \log f(\boldsymbol{v}, t),$$

where f is the distribution function of the velocity of a gas. For a monatomic gas of N atoms H can be written

$$H = -\tfrac{N}{V} \log(VT^{\frac{2}{3}}) + k,$$

whereas the entropy S of the same gas is given by

$$S = Nk \log(VT^{\frac{2}{3}}).$$

From these two equations Boltzmann was then able to derive an expression for S in terms of the H function.

$$S = -kHT.$$

An important feature of the H function is established in Boltzmann's H theorem:

$$\tfrac{dH}{dt} \leq 0.$$

Very soon afterwards, however, it was realized that the variation of the H function is invariant under the time reversal, and

$$\tfrac{dH}{d(-t)} \leq 0$$

too. In other words, the H function is constantly decreasing independently of the reversal of velocities of the molecules of the system. So according to the above connection between H and S, the entropy would increase independently of the direction of motion of the molecules.

The first to criticize this result was Loschmidt, who pointed out that the H theorem must be incorrect in the form given above. For it is impossible to derive any kind of irreversibility of the H theorem without smuggling in an element not in harmony with the reversibility of the collision between the molecules on which it is in fact based. One should therefore expect that for every set of motions which decreases H, there would be a corresponding reversed set of motions which increases H, and hence a set of motions which would give rise to a falling entropy S. A second criticism was raised by Zermelo based on the recurrence theorem proved by Poincaré. He argued that any permanently thermally isolated system must occupy its initial state infinitely many times. Such a thermodynamic system must therefore be subject to increasing as well as decreasing entropy throughout its existence. So if these objections are correct and if time's arrow can be explained by the second laws of thermodynamics, then the development of an isolated system could take place in either direction of time with equal probability.

The problem with Boltzmann's derivation of the H theorem lies in one of the basic assumptions concerning molecular chaos. He had simply assumed that the positions and velocities of particles were uncorrelated before they collided but not after. This made the H theorem asymmetric in time in conflict with mechanical laws. Correct as they were, the objections against Boltzmann's work nevertheless stood in stark contrast to the actual development of thermodynamical systems, all which move towards a state of increasing entropy. Since there is no help to be gained from the second laws of thermodynamics if it is to be understood in the light of statistical mechanics, Reichenbach attempted to solve the apparent inconsistency by introducing an asymmetry in the boundary conditions. His suggestion is that nearly every thermal system is a branch system. Such a system is never actually isolated long enough to move both towards increasing entropy as well as decreasing entropy. And being a branch system also explains why we see systems which only move towards higher entropy. A branch system is a system that branches off from its surrounding systems after they have interacted, whereupon it stays quasi-closed for a longer or shorter period of time. The conditions for being a branch system are:

(i) through interaction with the surroundings the system is brought into a state of low entropy and after being isolated it starts to move towards a state of higher entropy;
(ii) the microstates that provide the initial entropy on the macroscopic level should be randomly distributed among all possible states.

From statistical mechanics and these two conditions one can infer that nearly every branch system will develop towards a state of maximal entropy, and will stay there for a very long, but finite, period of time. Reichenbach, and later Grünbaum, conclude that these *de facto* irreversible systems can be used to pick out an objective direction of time.

Now time's arrow and the entropic development of the majority of the branch systems may coincide by chance and not as a matter of principle. In particular, if laws of nature don't favor a certain direction, it would be natural to draw the conclusion that *de facto* irreversible processes cannot be used as evidence for the existence of a temporal direction, and much less can they be used to define time's arrow. For time to be ontologically equivalent to physical processes it requires a *proof* that time's arrow and *de facto* irreversible processes coincide as a matter of principle. Thus, the relationist programme of reducing the temporal direction to asymmetric causal relations seems to run into deep troubles. As a way out the relationist might claim that time in the Universe actually has a direction. He can here invoke the strong cosmological principle and the assumption that the correct cosmological theory is a no boundary conditions quantum gravity theory. For it may be correct, as argued by Hawking (cf. [17]), that there exists a cosmological direction of time because (i) the initial quantum fluctuations were amplified by gravitational forces in the early universe in an highly orderly state so that the universe became lumpy and disordered while expanding, and (ii) disorder will continue to increase during a possible contraction of the universe. Such an origin would not permit local fluctuations with a reversed thermodynamic gradient. The cosmological direction of time is thereby completely dependent on the thermodynamic direction of time, and this explains the psychological asymmetry of time.

Originally Hawking believed that the no boundary model entails that if the expanding universe were one day to start contracting, then entropy would begin to decrease instead of going on increasing. In that case the order with which the Big Bang started out would be regained when the universe comes to an end in a Big Crush. Such an idea, which was not new, leads however to many paradoxes. Hawking now thinks he was wrong. This has led him into a new problem, because the no boundary model, as Huw Price (cf. [36]) first pointed out, apparently cannot explain why there is order at one end and disorder at the other, if the beginning and the end of the Universe are described by the same set of equations requiring no boundary conditions. No boundary conditions must exist in both directions of the initial state and the final state of the Universe. It seems that Hawking *presupposes* a temporal asymmetry where he should have explained it. Price himself, therefore, favors Hawking's old view in spite of all the paradoxes. A more daring conclusion might be that the relationist's idea of grounding a temporal asymmetry in entropy is completely wrong, and the relationist will always fail to explain temporally asymmetric relations on the basis of entropic features of the various laws of motion.

The substantivalist, on the other hand, could not consider *de facto* irreversible processes as a physical manifestation of time's arrow. He needs some further argument because the time invariant of laws of nature apparently counts as even stronger evidence against an inherent direction of time. Here Ilya Prigogine (cf. [39]) has produced an argument that, if it is sound, would prove the existence of substantival time and the objectivity of becoming. He holds that the fact that certain thermodynamic systems have a dissipative structure can be explained, but only if the past, the present, and the future are essential temporal features, and only if the past is ontically different from the future. A dissipative system is an open system very far from its thermal equilibrium. It is characterized by fluctuations around an initial point in the phase space, fluctuations that can produce qualitative changes from previously unorganized states into organized states. These changes cannot be calculated on the basis of the laws of the original state because stable and non-stable initial values will be chaotically mixed with each other, making it impossible to connect them with final values via determinate trajectories. Consequently, according to Prigogine, the distinction between laws and initial conditions of such systems is not maintainable in any strict way.

In the attempt to overcome the dichotomy between the fundamental reversibility on the microscopic level and *de facto* irreversibility on the macroscopic level, Prigogine therefore introduces a microscopic entropy operator M. It has the property that it does not commute with the Liouville operator L that describes the dynamical behavior of a system by acting on the density matrix ρ. Furthermore, M is seen as a product of an internal time operator T and its Hermitian conjugate $T\dagger$, where T is defined in terms of a nonunitary transformation operator Λ, and at the same time M is associated with the macroscopic entropy as this is expressed by a Lyapounov function containing a new density distribution $\tilde{\rho}$, which is obtained by letting Λ^{-1} act on ρ. Such a function is applicable, if the system is not describable by well-defined trajectories but only by a statistical distribution function, and this is in fact how dissipative systems can be described. As a result Prigogine states the law of entropy for dissipative system as

$$\frac{d\Omega}{dt} = -tr\,\rho\dagger(0)e^{iLt}i(ML - LM)e^{-iLt}\rho(0) \leq 0,$$

and by introducing a time evolution operator $\Phi = \Lambda^{-1}L\Lambda$, this equation can be written as

$$\frac{d\Omega}{dt} = -tr\,\tilde{\rho}\dagger(0)e^{i\Phi\dagger t}i(\Phi - \Phi\dagger)e^{-i\Phi t}\rho(0) \leq 0\ .$$

It is because of the nonunitary of the transformation operator that the temporal symmetry is broken in the description of these systems.

Nevertheless it is highly doubtful that Prigogine has shown all that he said he would. Granting that his analysis is correct, he has still not shown that dissipative structures provide us with evidence that time has an intrinsic direction. Neither does Prigogine prove that time really contains something like the past, the present, and the future, and that being future — apart from being perhaps epistemically different — is ontically different from being past in the sense that future events are ontologically indeterminate whereas past and/or present events are ontologically determinate. Thus, his argument cannot be taken as an argument for substantival time, nor for the existence of objective becoming. Also it is at least debatable whether dissipative systems, based upon some laws of nature, can count as empirical evidence for physical processes containing more than an actual arrow of time. For though Prigogine argues that we cannot make a clear-cut distinction between their laws of nature and their initial conditions, he also states that the time symmetry of dynamics can be broken in two ways: one in which the equilibrium is reached in the 'future', the other in the 'past' ([39], p. 212). If that is the case, it must be a mere fact of nature in which direction of time the symmetry is broken. Thus, we are no better off with dissipative systems than with branch systems.

2.4. *The Relativistic Argument against Becoming*

The special theory of relativity gains distinction first and foremost because of its replacement of the old idea of absolute simultaneity. Often the denial of absolute simultaneity has been considered as an argument against substantival time. But an unsound argument does not become sound by being repeated many times. Relative time is not the same as relational time. What frame-dependent simultaneity might prove is that future time instances are definite and, thus, future events are real; and, therefore, that there are no ontic differences between events in the past, the present and the future. It might prove that events are ontologically determinate regardless of their being past or being future, because two causally unconnectable events a and b may be simultaneous for one observer O, whereas for another observer O^*, a and b are not simultaneous. Thus, while a and b are both present to O, a may be present to O^* at a time where b may still be in the future of O^*. In contrast to a non-relativistic time where an alleged transition from indeterminateness to determinateness of events in the present mirrors the intrinsic order of time, relativistic time does not permit any event to be other than ontologically determinate. What that proves, if it proves anything, is not that time is parasitic on physical events and processes, but that features of pastness, presentness and futurity (defined in terms of determinateness) do not reflect an intrinsic order of time. There is no room for objective becoming in the theory of special relativity. The substantivalist could nevertheless argue that instants of time and durations are not ontically reducible to physical events or processes.

The above argument against real becoming and for the ontological determinateness of future events is due to Putnam (cf. [42]). Over the years several authors have pointed out that this argument goes through only if the assumption that a and b are equally real relative to O, because they are simultaneous, can be extended to include observers like O^* for whom b is in the future while a is present. Contrary to this, one could argue that reality can only be ascribed by a single observer relative to 'here and now' on its worldline. A slightly different approach to Putnam's argument is taken by Howard Stein ([47]). He argues that the structure of Minkowski space-time is compatible with the ontological indeterminateness of future events which is necessary for some realist interpretations of quantum mechanics and for objective becoming. Assuming that *here and now* is the space-time point that separates the determinate from the indeterminate, Stein defines a becoming relation Rab in terms of the geometric structure of the space-time as being invariant under suitable transformations of space-time itself:

(i) The state at any space-time point a is determinate as of a itself;
(ii) If the state at any space-time b is determinate as of a, then whatever is definite of b is also determinate as of a.
(iii) For any a, there is a space-time point c whose state is not determinate as of a.

Finally, Stein argues that space-time points b satisfying Rab all belong to the past lightcone of a.

Clearly, Stein is not attempting to prove the reality of objective becoming. By presupposing that there are future space-time points whose state is indeterminate as of a, he merely argues that an open future is compatible with relativity theory. It is, nevertheless, debatable whether Minkowski space-time permits in any convincing way ontologically indeterminate events or objective becoming as long as Stein's basic notion is *causal connectability* between an event occupying a single space-time point and events occupying past space-time points of this event. It also seems that even if such a notion of becoming makes sense, undesirable ontological implications about the reality of space-time points and of the intrinsic order of space-time itself follows from this. The reason is that two single events a and a^* can be space-like separated and therefore do not share a common lightcone. Consequently, events would change their status of being determinate to being indeterminate, or vice versa, relative to different observers in a and a^*, and the space-time points would change from being real to unreal, or vice versa, respectively.

A general conclusion must be that the issue of substantival or relational time remains quite open. Both positions have to address two usually related but independent questions: (i) Are instants of time and durations parasitic on physical events and processes or not? (ii) Does time have an intrinsic orientation independently of causal processes or not? Our understanding of physics does not give us the final answers yet.

3. THE LOGIC OF TIME

Recent years have seen a growing interest on the part of logicians in the semantics of tensed sentences, and among computer scientists in modelling temporal reasoning in connection with artificial intelligence systems. The basic concern of tense logic is the representation of temporal reasoning in its various forms. The idea is to develop and discuss formal languages that either mirror the temporal structure of natural languages or describe the way in which temporal elements can appear in and constrain cognitive processes. Time can be represented by different logics. The most common form of representation is the one which forms an essential part of our language as a tool for describing and discussing the course of events. We acquire some knowledge of time when we learn to master a language correctly, and through the study of the temporal structure of language we can get some useful information about the concept of time which a competent speaker implicitly presupposes.

3.1. *Motivation*

Philosophers have, of course, had much to say about the nature of time, and mathematicians and physicists have a lot to add from their perspective. More recently, linguists have also become interested in the temporal constructions of natural language. Can, then, a logician add anything of value to all this wisdom? Johan van Bentham's monograph on *The Logic of Time* ([48], p. xi) set out from this rather rhetorical question a dozen years ago. In the meantime not only psychologists, computer scientists and others have expressed their view on the subject and their knowledge about it, but logicians have also provided something new.

Temporal logic (or *tense logic* or *the logic of time*) is one of the most dynamic parts of nonclassical logic. As is usual in the case of a rapidly expanding discipline there is no unanimity among the specialists concerning what it is about and what specific area of research it covers. The logic of time surely has to do with the question of how formal counterparts of temporal relations interact with other logical connectives and operators. However, contemporary research on temporal logic and related disciplines and the manifold of contributions to it can hardly be surveyed in one single essay. Time is too fascinating a subject for logic and logicians to refrain from analysing and modelling it. There is a pressing need from specialists of other disciplines for mathematical structures framing their intuitions about time as well as moulding temporal aspects of the terminology they use. It is very likely that a logician working in nonclassical logic, computer science, mathematical linguistics or mathematical foundations of Artificial Intelligence will encounter these matters someday. John P. Burgess (cf. [8]) mentions several classes of motives for developing an autonomous tense logic:

1. *philosophical*: Contrary to our ordinary language, the language of mathematics and physics is untensed. Tense logic should teach us how to avoid confusing the tensed and the tenseless, and thus it has at least a 'therapeutic' value;
2. *exegetical*: It is tempting to look at the work of ancient writers through the spectacles of modern logic;
3. *linguistic*: The motivation flowing from that source is quite obvious. However, one should emphasize together with Burgess that tense logic does not attempt the faithful replication of every feature of deep semantic structure (and still less of the surface syntax) of natural languages; rather, it provides an idealized model giving the sympathetic linguist food for thought (cf. [8], p. 95);
4. *inter-logical*: Tense logic has important links with other parts of intensional logic, such as modal logic, deontic and causal logic;

5. and from *computer science*: Temporal operators may be used to express properties of programs (termination, correctness, safety, deadlook freedom etc.) and thus may be helpful in program verification.

All these motives result largely in varying approaches and lines of research and in numerous topics, which are sometimes barely compatible with each other. Nowadays, there are four main areas of research in temporal logic:

i) temporal logic in the narrower sense: investigations concerned with the usual questions of formal logic about calculi expressed in some specific temporal language: axiomatization, completeness, decidability, proof theory. Moreover, interrelations of temporal logic to other systems, e.g. modal and causal logic, higher order systems, deductive reasoning, model theory, theorem proving, as well as questions of mathematical linguistics such as the expressive power of temporal languages;
ii) modelling of natural language reasoning involving temporal terminology (tense and aspect);
iii) applications in computer sciences and Artificial Intelligence (planning problems, organization of time sharing processes, knowledge revision);
iv) philosophical applications.

As any scientific discipline with the proper self-esteem, temporal logic successfully tries to trace the lines of its tradition back to ancient roots. Aristotle's future sea-battle, Diodoros Cronus' master argument, or permanent trouble with temporal aspects of causal relations are early examples of time related metaphysical problems raising issues of formal reasoning about time. All these examples are in fact linked to ongoing research in multivalued, modal and causal logic.

3.2. *Ancient Roots*

As we saw earlier, Aristotle's famous ninth chapter of his work *On Interpretation* focused on the debate about truth, time and existence, and it has much a considerable impact on further investigations (it influenced both Jan Łukasiewicz's and Arthur Prior's work on tense logic). Earlier explications of the argument are closely connected to certain basic understandings of what tense logic is: Rescher ([43]) gave a well-known analysis of Aristotle's argument, Andersen and Faye ([2]) put forward a different interpretation. It turns out that these interpretations can be related to Prior's so called *Ockhamist* and to the *Peircean system*, respectively; i.e., to Prior's two major systems of indeterministic tense logic.

Diodoros Cronos' so-called master argument has the form of a trilemma. It claims that the following three propositions cannot all be true:

1. Every proposition about the past is necessary.
2. An impossible proposition cannot follow from (or after) a possible one.
3. There is a proposition which is possible, but neither is nor will be true.

Almost nothing is known about the course of the argument, nor about its author's intentions. Was it meant to be a case for determinism? Or is it concerned with the relations between time and modality? Whatever the answer to these questions, it proved to be extremely stimulating for further work in temporal logic.

It was the skeptics who drew attention to the temporal aspect of causal analysis, which previously ran happily without it. Does the cause precede its effect in time? If the answer is 'yes', what happens in between? If not, how can the world last, instead of collapsing into one single moment? Once such questions were raised, later generations of philosophers ensured that they were not to disappear.

3.3. *The Middle Ages*

In the Middle Ages, the concept of time plays a significant role in all major components of Medieval logic, i.e. in the theory of terms, the theory of entailment, of modal concepts and generally in nearly every philosophical consideration bearing on logic and language.

Scholastic logic was concerned with sophisticated logical analysis of natural language, and with tensed sentence forms in particular. To take an example, the belief in the Virgin Birth is expressed in different ways by the prophets ('Christ will be born'), by Jesus' contemporaries ('Christ is born'), and by the faithful in succeeding centuries ('Christ was born'). Do all these statements refer to one and the same object of faith, to an unchangeable entity? Or are they significantly different, although in some sense equivalent in meaning? Thomas Aquinas tried to mediate between these two approaches, called the 'res-theory' and the 'enunitiabile-theory' in *Summa theologiae*. It is a little exaggeration to claim that the most famous minds of that period, e.g. Boethius de Dacia, Paul of Venice (his *Logica Magna* was one of the representative positions of medieval logic), John Buridan, Walter Burleigh etc., were entirely concerned with reflections bordering on tense logic.

Arthur Prior once noted that claims like 'Tense distinctions are a proper subject of logical reflection' and 'What is true at one time is in many cases false at another time, and vice versa' (cf. [41], p. 104) are typical tenets of Medieval tense-logic. Øhrstrøm and Hasle (in [55]) comment, among many other things, also on that opinion of Prior. They emphasize the importance of the medieval period of logic, particularly because of the role of the concept of time plays in logic, and conclude that considerations of the temporal content were an integral part of logic in ancient and medieval times, but these topics were removed from the realm of logic during the renaissance (cf. [55], p. 118).

3.4. *Modern Times*

Time was finally reintroduced into logic during the last century, however. Both George Boole and Charles S. Peirce recognized the importance of the matter. Jan Łukasiewicz and other Polish logicians were concerned with tense-logical aspects in their work. Hans Reichenbach's writings on tenses were to have a great impact upon later investigations in theoretical linguistics. In 1941 J. N. Findlay published in the *Australasian Journal of Philosophy* his "Time: A Treatment of Some Puzzles", an important forerunner of modern temporal logic which is sometimes even considered to mark its starting point.

The Famous Precurser

Although not a system of temporal logic, Łukasiewicz' three-valued logic is frequently mentioned in the present volume. This is because Łukasiewicz explicitly refers to the topics of future tense and temporal logics: Many-valued logics were designed by him to solve two problems — that of contingent future sentences and the closely related problem of determinism (cf. [26] and [27], reprinted in [29]). It is the interaction of time and modality which is in focus when he claims that he wants contingent future sentences to be as *yet undecided* because if one of them is true (false), then it is true (false) now — and therefore there is no chance of the fact in question not happening (to happening). Such an argument is blocked, by prescribing a third truth value, differing from true and false, which allows future sentences to be undecided *and* truth-functional operators of 'possible that' and 'necessary that' to be defined. The discussion leads Łukasiewicz to his famous System L_3 which we present here:

Negation:

$\sim$	
1	0
1/2	1/2
0	1

Implication:

$\supset$	1	1/2	0
1	1	1/2	0
1/2	1	1	1/2
0	1	1	1

Conjunction and disjunction are definable in terms of $\sim$ and $\supset$:
$A \vee B =_{def} (A \supset B) \supset B$ and $A \wedge B =_{def} \sim(\sim A \vee \sim B)$.

The following is an axiomatic formulation of L_3:

(A1) $p \supset (q \supset p)$
(A2) $(p \supset q) \supset ((q \supset r) \supset (p \supset r))$
(A3) $(\sim p \supset \sim q) \supset (p \supset q)$
(A4) $((p \supset \sim p) \supset p) \supset p$
(R1) $[A \supset B, A/B]$ (Modus Ponens)
(R2) $[A/A\{a/B\}]$ (Uniform Substitution).

Rescher and Urquhart (cf. [44]) mention a few more sources. Besides Reichenbach they emphasize the study of Stoic logic and of further historical material (as carried out e.g. by Kneale in [22], Mates in [30], Moody in [33]) and "above all the endeavor by the Polish logician Jerzy Łoś to devise a system of temporal logic ..." ([44], p. 12).

One might hesitate, however, to take the last remark literally. Certainly, Łoś's system L is an original and interesting axiomatic calculus which has the following axioms:

(L1) $R_t(\sim p) \equiv \sim R_t(p)$
(L2) $R_t(p \to q) \to (R_t(p) \to R_t(q))$
(L3) $\forall t: \ R_t(p) \to p$

and a single rule:

(R) From $\vdash T$ infer $\vdash R_t(T)$.

$R_t(p)$ reads as "p is realized at the time t". (We assume that t ranges over the real numbers — cf. [15], p. 542.) Unfortunately, it was published in Polish, i.e. in a rather uncommon language, not to mention that the source was scarcely detectable (cf. [24]). For that reason Prior, who claimed (in [40], p. 212) to know Łoś's work only through Hiż's (condensed and not too enthusiastic) review ([18]), can hardly be blamed.

An independent line of research began with the work of von Wright on a chronological "logic of change" in the sixtees (cf. [53], [54]).

Arthur Prior

According to what we have seen above it seems fair to say that temporal logic in its modern form started with Prior's work about forty years ago. The basic idea was to handle tensed propositions as propositional functions, using times as arguments. Temporal reasoning can be formalized either in the realm of tense logic, or within a logic of instants of time. Since this is in line with McTaggart's well-known distinction between the A series and the B series, we may talk about A–*logic* and B–*logic*, accordingly.

In the following, we shall present two of Prior's historical constructions. We start with Prior's calculus K_t, which is an axiomatic tense logical system, *i.e.* an example of an A–logic. Let L_t be the classical propositional language extended by two monadic intensional connectives $\Box_p$ and $\Box_f$, together with their counterparts $\Diamond_p$ and $\Diamond_f$, defined as $\Diamond_p H =_{df} \neg\Box_p\neg H$ and as $\Diamond_f H =_{df} \neg\Box_f\neg H$. $\Diamond_p H$ and $\Diamond_f H$ read as 'for some time in the past it is true that H' and 'for some time in the future it is true that H', respectively. The reading of $\Box_p$ and $\Box_f$ follows easily from the above explications: 'H always has been the case' and 'H always will be the case'.

The system K_t consists of all formulae of L_t provable by *Modus Ponens* $[H, H \supset F/F]$ and *Necessitation* for both $\Box_p$ and $\Box_f$ $[H/\Box_p H$ and $H/\Box_f H]$ from the following set of axiom schemata:

1. all tautologies of classical propositional logic;
2. $\Box_f(H \supset F) \supset (\Box_f H \supset \Box_f F)$;
3. $\Box_p(H \supset F) \supset (\Box_p H \supset \Box_p F)$;
4. $H \supset \Box_f\Box_p H$;
5. $H \supset \Box_p\Box_f H$.

Other A–logics can be derived from K_t by adding further axioms or rules to the list above.

Next we introduce an example of a B–logic. First of all, we have once more to extend the classical language. We need a set Θ of instants structured by a relation $<$. For any propositional variable p and any $\tau \in \Theta$, the formula $T(\tau, p)$ is read as 'p is true at τ'. T extends on compounded formulae:

(1)	$T(\tau, \neg H)$	$\equiv$	$\neg T(\tau, H)$;
(2)	$T(\tau, H \wedge F)$	$\equiv$	$(T(\tau, H) \wedge T(\tau, F))$;
(3)	$T(\tau, \Diamond_f H)$	$\equiv$	$\exists \tau_1 (\tau < \tau_1 \wedge T(\tau_1, H))$;
(4)	$T(\tau, \Diamond_p H)$	$\equiv$	$\exists \tau_1 (\tau_1 < \tau \wedge T(\tau_1, H))$;
(5)	$T(\tau, \Box_f H)$	$\equiv$	$\forall \tau_1 (\tau < \tau_1 \supset T(\tau_1, H))$;
(6)	$T(\tau, \Box_p H)$	$\equiv$	$\forall \tau_1 (\tau_1 < \tau \supset T(\tau_1, H))$.

The ordered triple $\langle \Theta, <, T \rangle$ thus defined is called a B–*logical structure*. An easy calculation yields the following **Theorem 1**:

If $H \in L_t$ is provable in K_t, then $\forall \tau : T(\tau, H)$ holds in any B–logical structure.

Axioms added to a specific A–logic correspond to properties of the relation $<$ in its correlated B–logic. For instance, all theorems of a system resulting from K_t by adding $\Diamond_f \Diamond_f H \supset \Diamond_f H$ are true in any B–logic with transitive relation $<$. Further operators can be defined. For instance,
$\Box H =_{def} H \wedge \Box_p H \wedge \Box_f H$ ('always H')
or
$\Diamond H =_{def} H \wedge \Diamond_p H \wedge \Diamond_f H$ ('sometime H').
However, not all temporal operators expressible in B–logics are definable in this way. $\rhd H$ (read as: 'H is going to be uninterruptedly the case for some time'), defined by $\exists \tau_2, \tau_3 (\tau < \tau_2 < \tau_3 \wedge \forall \tau_1 (\tau_2 < \tau_1 < \tau_3 \supset T(\tau_1, H)))$ is an example. It was constructed by Hans Kamp in [19] (cf. [8], p. 117). Moreover, Kamp's two operators $\Diamond_u$ ('until') and $\Diamond_s$ ('since') define every temporal operator on the basis of linear, dense, non-ending instant logic.

Kripke structures

Systems of tense logic are intensional calculi, *i.e.* their language contains non-truth-functional operators (the truth value of a formula formed by such an operator is not determined by the truth values of its subformulas.) The prototypes of semantics designed to deal with intensional systems are relational structures (so-called Kripke models), and they were developed for the purposes of modal logic in the fifties. In modal logic the specific linguistic extension consists in a pair of two monadic intensional operators $\Box$ (read as: 'it is necessary that') and $\Diamond$ ('it is possible that') which are interdefinable by $\Box H \equiv \neg\Diamond\neg H$. Recall that a Kripke model is a triple comprising a non-empty universe W of 'possible worlds' together with a binary relation R in W and a valuation of the elementary formulae. Recursively, this valuation uniquely extends onto all modal formulae. The clause for the necessity operator $\Box$ reads:

$$\Box H \text{ is true at } x \text{ iff for all } y \text{ such that } xRy : H \text{ is true at } y.$$

Varying the properties of R and the internal structure of W, e.g. by splitting W into a normal and a non-normal part of the universe, we can alter the properties of $\Box$, thus making different modal laws valid (see van Bentham's correspondence theory ([49]) for the details).

The same technique is also suitable for handling temporal languages, i.e. languages with temporal operators. Kripke models can play the part of an adequate semantics for systems of temporal logic. Instead of $\Box$ and its counterpart $\Diamond$, here again we have the temporal twins $\Box_p$ and $\Box_f$, together with their counterparts $\Diamond_p$ and $\Diamond_f$. The set W of possible worlds of 'temporal Kripke structures' is best conceived as a manifold of (possibly punctiform) time intervals, structured by the accessibility relation R. Thus in the limit case of time instants (zero-dimensional time periods) R is just the usual 'earlier-than' relation. A classical model is associated with each element of the universe W. A formula H is true in $x \in W$ means that H is true in the time period x. Consequently, $\Box_p H$ is true in x means that H is true in every time period y accessible from x, whereas $\Box_f H$ is true in x iff H is true in every time period y which has access to x.

Perhaps, in case of philosophically relevant systems of modal logic, intuitions concerning necessity and possibility are primarily expressed on the syntactic level, *i.e.* in the form of modal formulae considered to be true. On the basis of the emerging axiomatic calculus one subsequently looks for an adequate semantics, e.g. in terms of Kripke structures. In that sense the properties of the accessibility relation of such a semantic modelling are secondary. Not so in temporal logic — the intuitions about time seem to determine the properties of R more directly than in modal logic. Thus the normal way of reasoning in temporal logic is rather the opposite one: First we have the 'structure of time', *i.e.* the semantics, and next we try to find a complete axiomatic representation.

For the sake of simplicity, let us assume that the elements of the universe are time points rather than (extended) time periods. There are various interesting and philosophically well-motivated properties of R to be considered. The most 'natural' demand seems to be *linearity*: R is irreflexive, transitive, and comparable. Within that framework, various modifications are possible: density, continuity, and the question whether time has a beginning and an end, are some of them. John Burgess ([8], p. 102 ff.) invites the reader for *a quick trip through tense logic*. For an orientation in this central realm of temporal logic, the best idea is surely to join him in that flying tour.

Linearity of time, however, has at least two serious rivals: *cyclic time*, and *branching time*. Furthermore, these concepts may be combined in several ways, assuming, for instance, linearity in the past and branching in the future, whereas the structure as a whole is closed by identifying the last and the first time point — interpreted as the 'Big Crush' and the (next) 'Big Bang', respectively.

Cyclic Time

In [9], Wolfgang Degen mentions a connection between causal order and cyclic time. The argument runs as follows:

Suppose that the causal relation $H \rightarrow_c F$ ('H causes F') holds between individual states of affairs (not between types of states of affairs) and that $\rightarrow_c$ denotes so-called exact causation characterized by

1. H is sufficient for producing F;
2. no proper part of H is sufficient for producing F;
3. nothing properly containing F is caused by H.

$H \rightarrow_c F$ defined in this way fulfills the following properties:

i. for every H there is exactly one F such that $H \rightarrow_c F$;
ii. for every F there is exactly one H such that $H \rightarrow_c F$.

Together i. and ii. express the thesis that exact causation is a permutation of the set of all individual states of affairs. Permutations consist of cycles. A finite set of individuals allows finite permutations only. If $\rightarrow_c$ forms more than one cycle, then the respective world is made up of causally independent regions. They may well count as different worlds, since a world must be causally connected. Then the definition 'the time of H is later than the time of $F =_{df} H \rightarrow_c F$' leads to causally based cyclic time. Note that the assumption about time points rather then periods is not essential.

Branching Time

The idea of branching time is a relatively new one. It was formulated in a precise manner by Henri Bergson at the end of last century. Jorge Luis Borges' short story "The Garden of Forking Paths" (1941) contains a detailed description of the new model of time. As a formalized idea it appeared for the first time in 1958 in a letter to Prior from Saul Kripke.

There are some further approaches using branching time to throw light upon the essence of the causal nexus. The first one is due to Nuel Belnap (see [7]). Gambling with technically rather undemanding tools — *i.e.* some branching (causal) order of the usual point events — he quickly puts himself in a position to profoundly discuss advanced topics in causal analysis in physics. Belnap believes in the sense of an indeterministic, branching time structure without a *Thin Red Line*, *i.e.* without postulating an actual future, distinguished among the possible courses of the world. Franz von Kutschera pursues his analysis of causation in a similar framework of branching histories ([52]). He states that the concept of necessity employed is not a logical or nomological one, but that 'necessary' is rather "what is the case no matter what turn the future history of the world will take" (cf. [52], p. 563).

Mixed Time revisited

The idea of differentiating the logical structure of the 'past' from that of the 'future' leads to a natural modification of the Kripke-structures employed in the analysis. Why not distinguish one element x_0 of the universe, the *present* or the *actual time*, and make use of two binary relations on W: the relation $<$, on the one hand, for structuring the past of x_0, and the relation $>$ on the other hand, for structuring the remaining part, the future of x_0? This allows for a very elegant proof (although still too technical for this introduction) of Thomason's incompleteness result on temporal logic, **Theorem 2** (cf. [49], p. 223):

The tense logic axiomatized by
$\Box_p(\Box_p H \supset H) \supset \Box_p H$ *(Löb's Axiom),*
$\Box_f \Diamond_f H \supset \Diamond_f \Box_f H$ *(McKinsey Axiom),*
is incomplete.

3.5. *Temporalizing a Logic*

Performing temporal logic normally means constructing and formulating a logic capable of describing a temporal discourse. The issue can, however, be posed otherwise. Finger and Gabbay show in [13] that it is possible to add a temporal dimension to a given logic, to temporalize an existing logic. In order to do that they use a (class of) temporal propositional logic(s) $\mathcal{T}$ applying it to an arbitrary logical system $\mathcal{L}$ and thereby arriving at its temporalized version $\mathcal{T}(\mathcal{L})$, for which soundness, completeness, and decidability are shown. Here we follow Finger's and Gabbay's presentation.

Let the language of $\mathcal{T}$ consist of propositional variables, $\sim$ and $\wedge$, and the binary temporal operators S and U ('since' and 'until'). Beside other truth-functional connectives, we define $\top$ and $\bot$ in the usual manner, and standard temporal operators as follows:

$FA =_{def} U(A, \top)$	$GA =_{def} \sim F \sim A$
$PA =_{def} S(A, \top)$	$HA =_{def} \sim P \sim A$

Let T be a nonempty set of time points and $<$ a relation over it. Let g — the valuation — be a function assigning to every $t \in T$ a set of propositional letters. Then $\mathcal{M} = \langle T, <, g \rangle$ is a model, and $\mathcal{M}, t \models A$ reads: A holds over model M at t. For $\mathcal{M} = \langle T, <, g \rangle$, $\mathcal{F} = \langle T, < \rangle$ is its underlying *flow of time*. By definition, we have:

$M, t \models p$	iff	$p \in g(t)$, where p is a propositional letter
$M, t \models \sim A$	iff	if it is not the case that $M, t \models A$
$M, t \models A \wedge B$	iff	$M, t \models A$ and $M, t \models B$
$M, t \models S(A, B)$	iff	there exists an $s \in T$ with $s < t$ and $M, s \models A$ and for every $u \in T$, if $s < u < t$ then $M, u \models B$
$M, t \models U(A, B)$	iff	there exists an $s \in T$ with $t < s$ and $M, s \models A$ and for every $u \in T$, if $t < u < s$ then $M, u \models B$.

Let $\mathcal{K}$ be a class of flows of time, then A is valid over $\mathcal{K}$ ($\mathcal{K} \models A$), if for every $\mathcal{M}$ whose underlying flow of time is in $\mathcal{K}$ and for every $t \in T$, $\mathcal{M}, t \models A$. Finger and Gabbay suggest the following minimal axiomatization of $\vdash_{S,U}$ (the S, U-temporal logic over a $\mathcal{K}$):

A0	All classical tautologies
A1	$G(p \supset q) \supset (U(p, r) \supset U(q, r))$
A2	$H(p \supset q) \supset (S(p, r) \supset S(q, r))$
A3	$G(p \supset q) \supset (U(r, p) \supset U(r, q))$

A4 $H(p \supset q) \supset (S(r,p) \supset S(r,q))$
A5 $(p \wedge U(q,r)) \supset U(q \wedge S(p,r),r)$
A6 $(p \wedge S(q,r)) \supset S(q \wedge U(p,r),r)$
SUBST If A is an axiom, q a propositional letter and B a formula, then $\vdash A\{q/B\}$ (Uniform Substitution).
MP $[A, A \supset B/B]$ (Modus Ponens).
TG $[A/HA, GA]$ (Temporal Generalization).

Let $L = \langle \mathcal{L}, \vdash_L \rangle$ be a logical system, with $\mathcal{L}$ its language and $\vdash_L$ an inference system. Let $\mathcal{M}_L$ be a model of the system L, and $\mathcal{K}_L$ the class of all models of L. In order to obtain a language for the temporalized system $T(L)$, the alphabet of L is enlarged by the operators S and U. Further, the set $\mathcal{L}_{T(L)}$ of formulas of $T(L)$ contains all formulas of L, all S- and U-formulas defined in the usual manner, and their boolean combinations (for details see [13], p. 211).

Consider now a flow of time $\langle T, < \rangle \in \mathcal{K}$, and let g be a function mapping elements of T to models in the class of models of L. Then a model of $T(L)$ is a $\mathcal{M}_{T(L)} = \langle T, <, g \rangle$, and the semantics of the temporalized system is defined by:

$\mathcal{M}_{T(L)}, t \models \alpha$ iff $g(t) = \mathcal{M}_L$ and $\mathcal{M}_L \models \alpha$ where $\alpha \in \mathcal{L}_L$ and is not built up from other formulas by boolean connectives.
$\mathcal{M}_{T(L)}, t \models {\sim}\alpha$ iff it is not the case that $\mathcal{M}_{T(L)}, t \models \alpha$.
$\mathcal{M}_{T(L)}, t \models \alpha \wedge \beta$ iff $\mathcal{M}_{T(L)}, t \models \alpha$ and $\mathcal{M}_{T(L)}, t \models \beta$.
$\mathcal{M}_{T(L)}, t \models S(\alpha, \beta)$ iff there exists an $s \in T$ such that $s < t$ and $\mathcal{M}_{T(L)}, s \models \alpha$ and for every $u \in T$, if $s < u < t$ then $\mathcal{M}_{T(L)}, u \models \beta$.
$\mathcal{M}_{T(L)}, t \models U(\alpha, \beta)$ iff there exists an $s \in T$ such that $t < s$ and $\mathcal{M}_{T(L)}, s \models \alpha$ and for every $u \in T$, if $t < u < s$ then $\mathcal{M}_{T(L)}, u \models \beta$.

This logic can be axiomatized very elegantly by the axioms and rules of the choosen temporal system (depending on the concrete flow of time), together with an additional rule:

PRESERVE For every formula $\alpha \in \mathcal{L}_L$, if $\vdash_L \alpha$ then $\vdash_{T(L)} \alpha$.

Finally, we arrive at a temporalized version of the original system which is now embedded in the upper-level logic.

3.6. *Tense Logics and Other Logics*

As was shown above, tense, or temporal, logics are usually considered to be part of a larger family of modal logics — tense operators describe the usage of certain, temporal, modalities. In this sense, tense logics are *bi-modal systems* whose semantics and meta-mathematics are close to that of ordinary (alethic) modal logic. This fact raises several questions about the correct location of tense logics within the system of logic in general. We would like to concentrate here on two particular points: although classical propositional or first order logic is usually the base for systems of tense logic, it is equally possible to build tense logics based on non-classical logics; and the close kinship between alethic and temporal modalities holds of course in both directions — modal operators are definable in terms of tense operators.

Since different definitions of 'possibility' and 'necessity' can be added to different tense logics, the problem of defining modal operators is a double one: One has to choose an appropriate definition *and* an appropriate tense logic system in order to obtain a modal fragment which is a certain alethic modal logic. Without proof (they can be found in [46]) we just report the results of such a procedure.

There are two routine pairs of definitions, the Aristotelian and the Diodorean ones:

ARI $\Diamond A =_{def} PA \vee A \vee FA$ $\Box A =_{def} HA \wedge A \wedge GA$
DIO $\Diamond A =_{def} FA \vee A$ $\Box A =_{def} GA \wedge A.$

The intuitive differences are obvious. In the case of a time model branching towards the future and being linear with respect to the past (which is favored by many philosophers because of the *closed-past-open-future* assumption) it may be useful to employ the definition suggested by Smirnov:

SMI $\Diamond A =_{def} PFA$ $\Box A =_{def} HGA.$

Accordingly, $\Diamond$ of SMI expresses the idea that — interpreted on a connected graph linear towards the past and branching towards the future — alternative possibilities for what is happening now, distinct from what is happening now, may arise on branches which branched off before now from the actual course of events.

Let K_t be Lemmon's well-known system based on classical propositional logic and formulated as follows:

$G(A \supset B) \supset (GA \supset GB)$ $A \supset HFA$

$H(A \supset B) \supset (HA \supset HB)$ $\quad A \supset GPA$
G- and H-Generalization $\quad$ Modus Ponens.

Other tense logical bases for defining modalities are then introduced as:

$K_c \;= K_t + HA \supset HHA$
$K_b \;= K_c + (PA \wedge PB) \supset (P(A \wedge B) \vee P(A \wedge PB) \vee P(PA \wedge B))$
$CL \;= K_b + (FA \wedge FB) \supset (F(A \wedge B) \vee F(A \wedge FB) \vee F(FA \wedge B))$
$SL \;= CL + GA \supset FA$, and $HA \supset PA$
$PL \;= SL + PA \supset PPA$
$PCr = K_c + GA \supset A$, and $GA \supset HA$,

where K_c explicates 'relativistic causal time', K_b — 'branching time', CL — 'linear time', SL is characterized by a lack of beginning and end of time, PL is Prior's logic (with 'dense time'), and PCr (also Prior's) models 'circular time'.

According to Smirnov (cf. [46], p. 24), several different standard systems of alethic modal logic can be obtained as fragments of these tense logics with the help of ARI, DIO, and SMI. To all of them except PCr, $HA \supset PA$ must be added, and then the following table shows which modal logics are obtained:

	DIO	ARI	SMI
K_t	T	B	B
K_c	$S4$	B	?
K_b	$S4$	B	$S5$
CL	$S4.3$	$S5$	$S5$
SL	$S4.3$	$S5$	$S5$
PL	$S4.3$	$S5$	$S5$
PCr	$S5$	$S5$	$S5$

The basic temporal part of the following construction corresponds to Lemmon's K_t, too. However, we are now going to consider the issue mentioned first, and provide an example of a tense logic based on a non-classical, paraconsistent, core. Priest introduces in [37] the semantics and a natural deduction system of what he calls a *dialectical tense logic*. Here we present the semantics only, referring the reader for the proof theory and an interesting application (the dialectical, tense logical interpretation of Leibniz' principle of continuity) to [37].

The language L of the system contains all formulas obtained from a set of propositional letters (S) by means of $\sim$, $\wedge$, $\vee$, P, F in the usual way. Truth values are the sets $\{0\}$, $\{1\}$, and $\{0, 1\}$, V being the set of truth values. Let X be a nonempty set, $<$ a relation over X, and v^i a function with domain X such that for all $x \in X$, $v^i_x : S \to V$; $\langle <, v^i \rangle$ is called an *interpretation*. Given an interpretation, v^i can be extended to an evaluation v by $v_x(a) = v^i_x(a)$ for all $a \in S$ and

(1a) $1 \in v_x(\sim A)$	iff	$0 \in v_x(A)$
(1b) $0 \in v_x(\sim A)$	iff	$1 \in v_x(A)$
(2a) $1 \in v_x(A \wedge B)$	iff	$1 \in v_x(A)$ and $1 \in v_x(B)$
(2b) $0 \in v_x(A \wedge B)$	iff	$0 \in v_x(A)$ or $0 \in v_x(B)$
(3a) $1 \in v_x(A \vee B)$	iff	$1 \in v_x(A)$ or $1 \in v_x(B)$
(3b) $0 \in v_x(A \vee B)$	iff	$0 \in v_x(A)$ and $0 \in v_x(B)$
(4a) $1 \in v_x(PA)$	iff	$\exists y < x \; 1 \in v_y(A)$
(4b) $0 \in v_x(PA)$	iff	$\forall y < x \; 0 \in v_y(A)$
(5a) $1 \in v_x(FA)$	iff	$\exists y > x \; 1 \in v_y(A)$
(5b) $0 \in v_x(FA)$	iff	$\forall y > x \; 0 \in v_y(A)$.

In the classical case, that is for $V = \{\{0\}, \{1\}\}$, we obtain the truth conditions for K_t mentioned above. The (b) cases are then redundant. The system is paraconsistent because $A \wedge \sim A$ is not necessarily invalid any longer, and Priest uses it for an argument in favour of the existence of real inconsistencies during (or at) *moments of change*.

As in nearly every other part of modern logic, temporal logic too, tries to absorb and take advantage of ideas from other disciplines. There are mutual stimulation from theoretical linguistics and cognitive psychology. One of the most promising realms is undoubtedly Artificial Intelligence. During the last decade this discipline has invented highly original and increasingly technical approaches (see e.g. [45]). It is enough to look through some recent volumes of logical or philosophical journals to see that temporal logic is in a state of accelerated dynamic growth, as Gabbay and Ohlbach put it (cf. [14], p. vi). Their *Proceedings of First International Conference on Temporal Logic*, although slightly biased by approaches from computer science, provides evidence for the impressive present state of this subject.

Indeed, previously logic added a lot to "all the wisdom about time", and it still has much more to say. The same is surely true for philosophy, linguistics, and every other discipline concerned with this issue. Time is ubiquitous, as Øhrstrøm and Hasle remark. It falls within the sphere of interest of many disciplines. Naturally, the result of the active research on time, tenses and temporal structures in the relevant sciences will be the better, the closer they collaborate and share their points of view.

4. ABOUT THE CONTRIBUTIONS

In his paper *Three Views of the Relationship Between Time and Reality*, Mauro Dorato distinguishes between an ontological and a semantical formulation of how time and reality are related. Focusing on the ontological formulation, he then discusses three different views of an open future: the instant view of reality in which there is no ontic difference between the open past and the open future; the empty-view in which the open future, contrary to the fixed past, contains no entities at all; and the half-full view in which causally determined entities in the future are real. The second and the third view are considered to ground all temporal asymmetries. Dorato concludes that even though most of the objections that have been traditionally levelled against the non-full view can be answered, the empty view with its global asymmetry would be preferable. He admits, however, that whether or not such a radical form of indeterminism can be justified depends on physical theories.

Lars Gundersen's paper *On Now-Ambiguities* supports a temporal interpretation of possible worlds. Now-ambiguities arise where natural language does not provide us with exact knowledge about the scope of 'now' and 'possible', for example. Gundersen's *World Model*, consisting of time fibres forming a rope which one can cut into *moments*, leads to a very intuitive semantics. This allows for covering both the temporal as well as the non-temporal aspect of the problem.

Uwe Meixner believes that McTaggart's attack on the reality of time is simply a result of a confusion of two possible meanings of the occurring temporal predicates in question. In his *The Objectivity of Time-Flux and the Direction of Time*, he gives a consistent description of the *flow of time* in terms of predicates running over events in a setting of classical logic and tense operators. This allows Meixner to obtain a number of intuitively well-founded results in a 'McTaggartian' style, concerning the presentness of events and their position in A series and in B series. In a final section he argues for a direction of time depending on the flow of time, thereby supporting the reality of tenses and becoming.

Fleeting Things and Permanent Stuff: A Priorean Project in Real Time is written by Paul Needham. Needham seeks a solution to a Priorean problem: things are made of *bits of stuff* — what is the precise logic of this process? His basic notion is a temporalized *composition relation*, philosophically supported by a commitment to the reality of time. Needham discusses the influence of modal operators in detail.

His analysis of Coming-to-be and Passing-away of things made of stuff culminates in a case study of 'burning a piece of paper'.

In *Existence in Time: From Substance to Process* Johanna Seibt formulates six presuppositions of the substance-ontological tradition. She sketches a process-ontological framework in which all these presuppositions are abandoned. This framework allows for an account of persistence which is shown to avoid the pitfalls of both the endurance and perdurance views.

In *Direction of Time: A Problem of Ontology, not of Physics* Erwin Tegtmeier starts his argumentation concerning the direction of time with an analysis of the direction of motion. The relation *earlier than*, which is asymmetric and transitive, together with the orderings of arguments of relational sentences, are the ontological foundation of the direction of time. With these tools he distinguishes (ontologically!) *earlier than* from *later than*, the former being existent, and the latter merely defined. Finally, Tegtmeier discusses attempts to reduce the ontological solution to that of a physical process.

Max Urchs' discussion in *Tense and Existence* concerns the interaction of time and existence. Temporal realism often seems to force fatalistic beliefs upon us. After examining Faye's and Kotarbiński's suggestions, he proceedes to his own solution of the matter. This is based on a conception of events as depending on an epistemic frame against which sentences have to be evaluated. This makes possible the *breaking off* of causal chains whose members belong to different frames, and so for Urchs not only is the future open, but even the past can — partially, and in a sense — disappear.

The physics part contains a contribution of Andreas Bartels, *Do Times Exist? Two Conceptions of Reality for Instants of Time*. Here he asks whether instants of time are real in the sense of being actual, or whether they are real in the sense of being objective. The first question is answered in the negative because in a relativistic world the transition from definiteness to non-definiteness has no ontological counterpart. The second question on the other hand, is answered positively. Bartels alludes to Quine's criterion of existence in which values of a theoretical variable are real. He then shows that relativistic proper time fulfills this standard of objectivity.

Jan Faye's two contributions on the physical direction of time are closely related. In both papers he rejects what is called the standard interpretation: the view of which reversible processes do not provide us with a causal or a temporal asymmetry since these processes are T-invariant. Instead it is claimed that we need some kind of irreversible processes which are, at least, *de facto* not T-invariant. Irreversible processes may then be identified with increasing entropy or expanding, retarded waves. Faye maintains, however, that the standard view is wrong because it misses two fundamental distinctions: one between process tokens and process types, and another between the active time reversal transformation and the passive time reversal transformation. In his first paper *Is the Mark Method Time Dependent?* he argues, contrary to a recent paper of Wesley Salmon's, that a satisfactory physical account of causation will continue to elude us as long as the widespread belief that Reichenbach's mark method, or similar procedures, cannot inform us about the causal direction of reversible processes independently of a temporal orientation is maintained. In his second paper *Causation, Reversibility and the Direction of Time*, Faye explores the possibility of understanding the passive and active time reversal transformation physically in terms of positive and negative energy solutions.

Sometimes philosophers draw a line between physical time and subjective time, claiming that becoming is completely mind-dependent. Massimo Pauri argues in his paper *The Physical Worldview and the Reality of Becoming* that the reality of time and becoming is intrinsically interwoven with the mind-body problem. On the one hand, he contends that there is no room for objective becoming in the description of the world in terms of physical laws. On the other hand, he claims that the subjectivity of our sense experience cannot explain the phenomenological awareness of becoming which is intimately related to a similar awareness of the power of free will. Pauri therefore seeks to establish a third notion of time to the effect that real time has to be distinguished from both physical time and psychological time. Real time, according to him, consists in the real becoming, a transsubjective and emergent feature of the world into which physics cannot inquire.

In *Relations between Sets of Time Points and Quasi-Linear Orderings* Wojciech Buszkowski exploits algebraic means to settle the question of whether there is an alternative formal explication of the phrase "...before ..." in the form of an operator B which is constructed so as to agree with linguistic evidence. By assuming irreflexivity, he is able to give such a definition for finite universes of time points. Furthermore, he shows that so-called quasi–linear ordering is both necessary and sufficient for B's anti-additiveness. Similar results hold in infinite universes.

Linguistic and Tense Logical Considerations on the Generality of a Three-Point Structure is written by Per Hasle. Apart from historical remarks on the way in which Jespersen influenced Reichenbach's view on linguistics, Hasle examines a specific connection between some linguistic temporal categories and tense logic.

Karl-Heinz Krampitz, Uwe Scheffler and Horst Wessel, working together for quite a few years, were finally able to deal with the question of whether or not Socrates is mortal? In *Time, Truth and Existence* they outline their ideas on truth and existence in time against the background of a historical dispute between Kotarbiński and Leśniewski. Working within the Sinowjew/Wessel approach to the so-called non-traditional theory of predication, they elucidate central notions of ontologic (the formal study of ontology), such as existence in time.

Ingolf Max in *Dimensions of Time* uses the language of classical propositional logic which is extended by special operators to form pairs or triples of propositional variables and variable functors that take such two-dimensional or three-dimensional expressions as their arguments. The basic idea is to represent tense-phenomena within this framework without using an extra place for variables whose range is a set of instants of time. In spite of such an expressive power of this syntactically enriched classical propositional calculus there are theorems which demonstrate that, with regard to valid sentences, we have certain theorems which tell us that everything is reducible to the classical propositional calculus without any form of multi-dimensionality. Therefore, he suggests a formal explication of tenses in a classical style.

Yaroslav Shramko's *Time and Negation* starts with what he calls the mysterious aspects of time. He proceeds with a semantics which contains no explicit temporal elements which, nevertheless, turns out to be an appropriate framework for expressing ideas about time.

In *A New Tempo-Modal Logic for Emerging Truth: Prior's Idea of Non-Statability as a Solution to the Problem of Contingent Individuals* Mogens Wegener and Peter Øhrstøm concentrate on the idea of non–statability in order to disclose its importance for the logic and philosophy of time. Furthermore, they propose an alternative system W of temporal modal logic which they believe may compete favourably with the classical logic of non–statability. W is not only indeterministic in the narrower sense but it discards the idea of timeless truth by implying truth to emerge in time along with reality.

Peter Øhrstrøm's *A. N. Prior's Ideas on the Relation between Semantics and Axiomatics for Temporal Logic* concludes the volume. Øhrstrøm analyses the traditional distinction between A– and B–temporal logics: on the one hand there are ordinary tense logics, i.e. formal theories of tense operators, and on the other hand there is the logic of instants in terms of earlier–later calculi. They are considered as axiomatic or proof theoretical calculi, and as semantical systems, respectively. Øhrstrøm elaborates some ideas of Prior, who maintained that B–logic can be derived from A– logic.

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REFERENCES

1. H. G. Alexander, editor. *The Leibniz–Clarke Correspondence.* Manchester University Press, Manchester, 1956.
2. H. B. Andersen and J. Faye. Om fremtidige søslag. In *Studier i antik og middelalderlig filosofi og idéhistorie*, volume 40-43 of *Museum Tusculanum*, pages 149–181. Museum Tusculanums Forlag, København, 1980.
3. Aristotle. *The Works of Aristotle*, volume 8. Clarenton Press, Oxford, 1928. Physics.
4. Aristotle. De Interpretatione. In *Aristotle's Categories and De Interpretatione*, pages 43–68. Oxford University Press, Oxford, 1963. (Translated with Notes by J. L. Ackrill).
5. Augustine. *Confessions.* Henry Regnery, Chicago, 1948.
6. I. Barrow. *Geometrical Lectures: Explaining the Generation, Nature and Properties of Curve Lines.* London, 1735. trans. I. Newton.
7. N. Belnap. Branching space-time. *Synthese*, pages 385–434, 1992.
8. J. P. Burgess. Basic tense logic. In D. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic vol. II, Extensions of Classical Logic*, pages 89–133. Reidel, Dordrecht, 1984.
9. J. W. Degen. Tense logic. In H. Burkhardt and B. Smith, editors, *Handbook of Metaphysics and Ontology vol. II*, pages 888–891. Philosophia Verlag, München, 1991.
10. J. Earman. *World Enough and Space–Time.* A Bradford Book. MIT Press, Cambridge/Mass., 1989.
11. J. Faye. *Et Naturfilosofisk Essay om Tid og Kausalitet.* Jørgen Paludans Forlag, København, 1981.

12. J. Faye. *The Reality of the Future.* Odense University Press, Odense, 1989.
13. M. Finger and D. M. Gabbay. Adding a temporal dimension to a logic system. *Journal of Logic, Language, and Information*, 1(3):203–233, 1992.
14. D. Gabbay and H. J. Ohlbach, editors. *Temporal Logic. Proceedings of the First International Conference*, volume 827 of *Lecture Notes in Artificial Intelligence*, Berlin, 1994. Springer.
15. J. Garson and N. Rescher. Topological logic. *Journal of Symbolic Logic*, 33(4):537–548, 1968.
16. A. Grünbaum. *Philosophical Problems of Space and Time.* D. Reidel, Dordrecht, second enlarged edition, 1973.
17. S. W. Hawkins. *A Brief History of Time.* Bantam Press, London, 1988.
18. H. Hiż. Review of 'J. Łoś: Podstawy analizy metodologicznej kanonów Milla'. *Journal of Symbolic Logic*, 16(1):58–59, 1951.
19. H. Kamp. *Tense Logic and the Theory of Linear Order.* PhD thesis, UCLA, 1968.
20. I. Kant. *Critique of Pure Reason.* Macmillan, London, 1963. trans. N. Kemp Smith.
21. I. Kant. *Kritik der reinen Vernunft.* Verlag Philipp Reclam jun., Leipzig, 1979.
22. M. H. Kneale. Implication in the fourth century B.C. *Mind*, 44:485–495, 1935.
23. J. Locke. *An Essay Concerning Human Understanding.* Everyman's Library, London, 1961.
24. J. Łoś. Podstawy analisy metodologicznej kanonów Milla. *Annales Universitatis Mariae Curie-Skłodowska, Sectio F*, 2:269–301, 1948.
25. E. J. Lowe. McTaggart's paradox revisited. *Mind*, 101(402):323–326, 1992.
26. J. Łukasiewicz. O logice trojwartosciowej. *Ruch Filosoficzny*, 5:170–171, 1920.
27. J. Łukasiewicz. Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalküls. *CR Varsovie*, 23:51–77, 1930.
28. J. Łukasiewicz. On determinism. In L. Borkowski, editor, *Selected Works*, pages 113–121. North-Holland, Amsterdam / London / Warszawa, 1970.
29. J. Łukasiewicz. *Selected Works.* North-Holland, Amsterdam / London / Warszawa, 1970. (Edited by L. Borkowski).
30. B. Mates. *Stoic Logic.* University of California Press, Berkeley, 1953.
31. J. M. E. McTaggart. *The Nature of Existence.* Cambridge University Press, Cambridge, 1927.
32. D. H. Mellor. *Real Time.* Cambridge University Press, Cambridge, 1981.
33. E. A. Moody. *Truth and Consequences in Medieval Logic.* Amsterdam, 1953.
34. I. Newton. *Mathematical Principles of Natural Philosophy.* University of California Press, Berkeley, 1960.
35. W. H. Newton-Smith. *The Structure of Time.* Routledge & Kegan Paul, London / Boston / Henley, 1980.
36. H. Price. A point on the arrow of time. *Nature*, 320:181–182, 1989.
37. G. Priest. To be and not to be: Dialectical tense logic. *Studia Logica*, XLI(2-3):249–268, 1982.
38. G. Priest. Tense, *tense* and TENSE. *Analysis*, pages 184–187, 1987.
39. I. Prigogine. *From Being to Becoming: Time and Complexity in the Physical Sciences.* W. H. Freeman & Company, San Francisco, 1980.
40. A. Prior. *Past, Present and Future.* Oxford University Press, Oxford, 1967.
41. A. N. Prior. *Time and Modality.* Clarendon Press, Oxford, 1957.

42. H. Putnam. Time and physical geometry. *Journal of Philosophy*, 64:240–247, 1967.
43. N. Rescher. Truth and necessity in temporal perspective. In R. Gale, editor, *The Philosophy of Time*, pages 183–220. New Jersey, 1968.
44. N. Rescher and A. Urquhart. *Temporal Logic*. Springer, Vienna, 1971.
45. Y. Shoham. *Reasoning about Change. Time and Causation from the Standpoint of Artificial Intelligence*. The MIT Press, Cambridge/Mass., 1988.
46. V. A. Smirnov. Defining modal operators by temporal operators (In Russian). In A. S. Karpenko, E. A. Sidorenko, and V. A. Smirnov, editors, *Modal and Intensional Logics and their Application to Problems in Methodology of Science*, pages 14–31. Nauka, Moscow, 1984.
47. H. Stein. On relativity theory and the openness of the future. *Philosophy of Science*, 58:147–167, 1991.
48. J. van Benthem. *The Logic of Time. A Model-Theoretic Investigation into the Varieties of Temporal Ontology and Temporal Discourse*. Reidel, Dordrecht / Boston / London, first edition, 1983.
49. J. van Benthem. Correspondence theory. In D. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic vol. II, Extensions of Classical Logic*, pages 167–247. Reidel, Dordrecht, 1984.
50. P. van Inwagen. The incompatibility of free will and determinism. *Philosophical Studies*, 27:185–199, 1975.
51. P. van Inwagen. *An Essay on Free Will*. Clarendon Press, Oxford, 1983.
52. F. von Kutschera. Causation. *Journal of Philosophical Logic*, 22:563–588, 1993.
53. G. H. von Wright. *Norm and Action. A Logical Inquiry*. Routledge & Keagan Paul, London, 1963.
54. G. H. von Wright. And next. *Acta Philosophica Fennica*, 18:293–304, 1965.
55. P. Øhrstrøm and P. F. V. Hasle. *Temporal Logic — From Ancient Ideas to Artificial Intelligence*. Kluwer, Dordrecht / Boston / London, 1995.