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In recent years there has been a growing interest in the study of the logic of causal reasoning. It is generally agreed that the causal nexus in itself is not logical in nature. But what has a logical structure is the way we think and speak about causal phenomena. And because causal phenomena pop up everywhere and grasping them is so fundamental for the structuring of the world, causal reasoning plays a major role in everyday life and in the empirical sciences. While people are usually able to apply a causally colored terminology without major difficulties, a closer philosophical analysis often reveals a striking difference in the intuitions about causality and the inter–relationships between a huge family of uncertain terms, giving rise to a lot of conceptual tensions. It therefore seems valuable to try to use formal methods in order to unravel the inherent structure of our causal thinking. Hopefully, this will lead to a better understanding of our basic intuitions and to improve inferences based on causal arguments.

Formal Systems

In order to gain such an understanding one must first of all consider the question concerning what kind of logic represents our causal reasoning. During the past decades a large number of non–classical calculi have been developed, alongside classical logic, which may also be proper candidates for the formalization of causation. First, one has to decide whether one believes that first–order logic is appropriate or whether the analysis can be carried out within the confines of propositional logic. That is, one has to face the question whether to think about causation as a relation or as a sentential connective.

One may prefer sentential connectives to (two-argument) predicates as counterparts of causal relations. The reason for this is simply due to the following categorial ground: the causal nexus connects singular events. Such items could formally be represented quite naturally by propositional variables. So any such formalization of the causal nexus is supposed to deal with propositional variables. In case of a causal predicate the argument places are filled in with individual variables, *i.e.* formal objects representing (names for) individuals.

Perhaps it is always possible to translate one of these categories into the other. If so, it doesn't really matter whether one presents one's analysis in terms of sentential connectives or predicates. Sentential connectives as sentence–forming operators are the natural counterparts for the "... *because of*..." phrase, just as they are appropriate for the "*if*..., *then*...", or the "... *and*..." construction.

As a matter of internal logical aesthetics one sometimes has to lift a formal ap-

proach from merely sentential considerations to the first–order level in order to obtain a fully developed logical formalism. Then, however, one should think about a predicatively structured formalization of the notion of a singular event and end up with a second order theory.

In favour of the introduction of causal predicates one could mention causally colored secondary notions like "... brings about ... " or "... is responsible for ... ". There are two major arguments in favour of treating causation as a relation, both of them are concerned with ontology, and not with logic. The first deals with the kind of entities causation involves: usually the entities are taken to be events. Our talk about causal relations directly refers to this fact. Thus, causation is regarded as a relation between objective events, whereas an implication is a sentence-forming operator on pairs of sentences. Obviously one can either use other names of entities as arguments in causal sentences (tensed propositions, propositions expressing an event), or understand the sentences connected by the causal implication as sentences reporting the occurrence of events. In this latter case what then has to be solved is the question whether the occurrence of an event is an event identical with the original event. The second argument in favour of the relational account is connected with the nature of causality itself: while connectives, implications, are a matter of logic, relations and their relata are situated in the world of empirical things, they are matters of facts. Thus, realist arguments are on the side of a relational approach. Those who seek causality in the real world would be inclined to understand it as a relation.

The question of whether causation is represented by a sentential connective or a predicate raises a further problem: what can be regarded as an appropriate formalization and what cannot. There is no easy solution. The slogan is: "Take the definition of the causal nexus, formalize it and call what you obtain a 'causal connective or predicate'". However, this answer is not really helpful in the present case. The various explications of the causal connectives or predicates that occur in the literature are seldom precise enough to allow immediate logical formalization. Furthermore, although there are many definitions, there is, at the same time, a noteable lack of agreement on these definitions.

It is, indeed, a natural question to ask for more or less adequate formalizations of concepts of the causal nexus as they are elucidated by philosophers. There are a few examples in the literature. It might, however, be seen as much more important (and interesting as well) to formalize causal terminology which is in fact used in empirical sciences. In providing for such a formalization we must first of all ask for the object of formalization. How do we filter out the notion which is to be formalized from the maze of utterances in which it is buried?

One may try to overcome the difficulties obviously arising from that situation in various ways:

On the basis of more or less clearly formulated intuitions one defines formal objects and calls them "causal junctors", "causal connectives", "causal predicates", or whatever. Subsequently, these nominal definitions are to be justified by proving their adequacy to the terminology of causality functioning in real language, otherwise they are not justified at all. This procedure was chosen by Jan Łukasiewicz in his "Analysis and Construction of the Concept of Cause", one of the very first

papers on causal logic (cf. [14]). It seems, however, that this choice merely postpones the problem instead of solving it. The reason is that logic itself has almost no power to bring its constructions into any spoken language, *i.e.* to execute obedience to the rules concerning the use of its artificially linguistic creations. For if the intended users of the formalization reject the proposed metamathematical constructions as appropriate formal counterparts then the formalization turns out to be a failure.

2. Starting from the use of causal terminology in some specified realm of natural language (say, in a given empirical science) one should first establish as precisely as possible all the available properties of causal relations as used in the realm considered. Next, one has to formalize these properties. In this manner a frame of metamathematical properties is obtained which the formal counterparts of causal relations must possess. Finally, one is in a position to construct — so to say "in stock" — a manifold of connectives falling into this frame whose formal properties vary to some extend. One might thereby hope to cover all the intentions of the causal notions used in the texts considered.

All appropriate metamathematical counterparts of the kind of causation considered must fulfill the required frame conditions and may therefore be elements of the constructed manifold of connectives. So "all" what remains is to figure them out.

3. Starting from well-founded ontological assumptions concerning reality, one designs all possible kinds of causal connections (*i.e.* those consistent with the ontological settings about the structure of the world), and then distinguishes the cases of practical relevance, *i.e.* the kinds of causal nexus to be found in the real world.

This third variant could be called "formal–ontological" causal analysis. As Roman Ingarden puts it when speaking about causally structured worlds:

"The task of formal ontology is nothing but to give an overview of these possibilities. Only taking into account material ontology could possibly reduce the number of purely formally established possibilities, and only then metaphysics or the natural sciences might decide which one of these various cases is indeed realized." ([7], p. 390)

In the following we shall present various formal approaches for explicating causation. As the matter is technically complicated enough we shall usually follow the manner of presentation adopted by their respective authors. Although it is often possible to switch from the sentential to the first order level we will give the former version — in most cases the extension to the latter is obvious. The systematization of the various formal approaches presented below is neither exhaustive nor leaves out the possibility that some of these systems will fall into more than one group.

The First Attempt

The first well–known attempt at formally characterizing an explicitly *causal* connective was undertaken by Arthur Burks in 1951 (cf. [3]). It is based on modal logic and includes a causal necessity operator \square :

- 1. $(A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$
- 2. $A \supset (B \supset A)$
- 3. $(\sim A \supset \sim B) \supset (B \supset A)$
- 4. $(\alpha)(A \supset B) \supset (A \supset (\alpha)B)$, if α is not free in A
- (α)(A ⊃ B), where α is an individual variable, β is an individual variable or constant, and no free occurrence of α in A is in a well formed part of A of the form (β)C, and B results from the substitution of β for all free occurrences of α in A
- 6. $\Box A \supset \Box A$
- 7. $\Box A \supset A$
- 8. $(\alpha)(A \supset B) \supset ((\alpha)A \supset (\alpha)B)$
- 9. $\Box(A \supset B) \supset (\Box A \supset \Box B)$
- 10. $c(A \supset B) \supset (cA \supset cB)$
- 11. $(\alpha) \Box A \supset \Box(\alpha) A$
- 12. $(\alpha) \odot A \supset \odot(\alpha) A$
- I. From A and $A \supset B$ to infer B
- II. If A is an axiom, so is $(\alpha)A$
- III. If A is an axiom, so is $\Box A$

Based on the system, a causal connective is to be defined as follows:

Definition (Causal Implication)

A causally implies B, iff $c(A \supset B)$.

An interesting detail can be added: whereas much further research in causal logic after Burks starts with counterfactuals in order to define causality, Burks himself uses causal implication in order to define counterfactuals. The causal implication defined above is something between a strict implication and the material implication, and is the object of various criticisms.

The Counterfactual Analysis

The most representative advocate of the counterfactual approach to causality is probably David Lewis. The basic idea underlying the introduction of the counterfactual condition consists in the belief that the occurrence of a cause makes a difference in the world: the world would be different, if the cause had not taken place. Thus, one has to take into account not only the actual situation, but also one or more possible situations in order to evaluate a causal statement. Usually this is done in a possible worlds framework. In order to avoid misunderstandings we remind the reader of the following background aspects of Lewis' analysis:

1. Causes and effects are *events* in the everyday sense of the word.

- 2. The topic is *token* causation, that means, causation involving particular, singular events (and not classes or types of events).
- 3. To be a cause does not mean to be *The* single, unique, complete *Cause*; but rather to be one of the causes.
- 4. The following analysis is the analysis of *causation under determinism*.

Lewis presupposes a theory of counterfactuals. The truth conditions for counterfactuals are as follows:

 $A \Box \rightarrow B$ is true (in a world w) iff either

- i. there are no A-worlds; or
- ii. some A-world where C holds is closer to w than is any A-world where C does not hold.

The notion of "closeness" of worlds may have different formal and intuitive interpretations. Lewis, however, prefers to understand it in terms of a similarity relation with two restrictions: our actual world is closest to actuality (thus, from $A \wedge B$ follows $A \Box \rightarrow B$, sometimes called the non-connection thesis); and similarity generates a weak ordering of worlds (the axiomatization below and some results on counterfactuals are to be found in his [11], first published in 1973).

There are several axiomatic formulations of Lewis' ideas on counterfactuals. One of the earliest runs as follows:

- 1. All truth-functional tautologies are axioms.
- $2. \quad A \Box \!\!\! \rightarrow \!\!\! A$
- 3. $((A \Box \rightarrow B) \land (B \Box \rightarrow A)) \supset ((A \Box \rightarrow C) \equiv (B \Box \rightarrow C))$
- 4. $((A \lor B) \Box \to A) \lor ((A \lor B) \Box \to B) \lor$ $(((A \lor B) \Box \to C) \equiv (A \Box \to C) \land (B \Box \to C))$
- 5. $(A \Box \rightarrow B) \supset (A \supset B)$
- 6. $A \land B \supset (A \Box \rightarrow B)$
- I. If A and $A \supset B$ are theorems, so is B.
- II. If $(B_1 \land \ldots) \supset C$ is a theorem, so is $((A \Box \rightarrow B_1) \land \ldots) \supset (A \Box \rightarrow C)$.

Let A_1, A_2, \ldots and C_1, C_2, \ldots be families of propositions of equal size, no two members of which are compossible. Then the family C_1, C_2, \ldots depends counterfactually on the family A_1, A_2, \ldots , if all the counterfactuals $A_1 \Box \rightarrow C_1, A_2 \Box \rightarrow C_2, \ldots$ between the corresponding propositions are true. Let e be an event and O(e) be the proposition that holds at all and only those worlds where e occurs. This makes it possible to introduce event causation:

Definition (Causal Dependence I)

Let c_1, c_2, \ldots and e_1, e_2, \ldots be distinct possible events such that no two of the c's and no two of the e's are compossible. The family e_1, e_2, \ldots of events depends causally on the family c_1, c_2, \ldots of events, iff the family $O(e_1), O(e_2), \ldots$ depends counterfactually on the family $O(c_1), O(c_2), \ldots$

Definition (Causal Dependence II)

Let c and e be distinct possible particular events. Then e depends causally on c iff the family O(e), $\sim O(e)$ depends counterfactually on the family O(c), $\sim O(c)$.

Definition (Causation)

Causation Relation is the transitive closure of Causal Dependence II.

The major advantages of the construction above is the expression of a conception of causation, within a well-investigated and easy manageable formalization, which is non-committal on a number of issues such as backward causation and (indirect) self-causation are allowed, and even causation's antisymmetry as some examples show. Many philosophers find these possibilities counterintuitive, but they all fit Lewis' intuitions, and so he is not worried about them.

Other Modal Approaches

Modal logic is still the best developed area of non-classical logic. For reasons explained in the section "The axiomatic approach" causal analysis in modal logic is mostly performed by semantical means. The most convenient model-theoretical tool is undoubtedly Kripke's possible world semantics, and many considerations grounded on Kripke's framework are found in the literature.

Independently of Lewis a similar counterfactual approach was developed by Robert Stalnaker (who actually published the first paper using Kripke–models in causal analysis, see [23]) and by others. According to some authors ([6], [13]), it is even possible to pursue formal causal analysis completely within the realm of modal logic. One should, however, avoid defining the causal connective merely as some kind of necessitated implication. Otherwise, as Kit Fine had already remarked, one would be forced to accept some paradoxical consequences.

Besides conceptions based on Kripke–frames there are some other constructions making use of modal–logical techniques. Most of them rely on relational frame semantics. There are some exceptions based on neighborhood semantics (see for instance [22]), not to mention more general semantical types such as Boolean frames ([27]).

A quite original conception, remarkable both with respect to its age and scope, is due to Stanisław Jaśkowski. In his [8] he considers an extension FOR_f of the classical propositional language enlarged by one two–argument sentential connective \forall_f . Where H and G are formulas, $\forall_{f(H)}G$ is well–formed too and should be read: "G is true for all values of the factors of H". The factors of a formula H are the circumstances relevant for the state of affair described by H. The precise meaning of this notion is revealed in the subsequent semantical considerations. Jaśkowski constructs a causallogical sentential system Q_f via an interpretation of FOR_f by a bundle of translations $\{t_i\}_{i \in I}$ in first order predicate calculus PC_1 : $Q_f = \{H \in FOR_f; \text{ for all } i \in I :$ $t_i(H) \in PC_1\}$.

It turns out that the original construction of Q_f can be generalized considerably. Instead of PC_1 any regular modal system L may be used. One starts with the class \mathcal{K}_L of all f-o Kripke-frames $\mathcal{F} = \langle W, Q, R, P \rangle$, for L, *i.e.* $\mathcal{F} \models_m L$. Then \mathcal{K}_L is adequate for $L: \mathcal{E}(\mathcal{K}_L) = L$. Next one forms products of \mathcal{K}_L -frames, so-called n-dimensional

frames. The notion of a *n*-dimensional model and the acceptance relation between *n*-dimensional models, points of such models and formulas of FOR_f are explained as usual. The only interesting case is the interpretation of the non-classical connective. Let $\mathcal{M}^{(n)} = \langle W_1 \times \ldots \times W_n, \{\mathcal{R}_i\}_{i \in I}, \{\mathcal{Q}_i^{(n)}\}_{i \in I}, \mathcal{P}, \phi \rangle$ with $\mathcal{Q}_i = W_1 \times \ldots \times W_{i-1} \times Q_i \times W_{i+1} \times \ldots \times W_n, \mathcal{R}_i = id_1 \times \ldots \times id_{i-1} \times R_i \times id_{i+1} \times \ldots \times id_n, \mathcal{P} = \Pi_1 \times \ldots \times \Pi_n$ and $\phi : FOR_f \longrightarrow \mathcal{P}$ be a *n*-dimensional model, $x^{(n)} = (x_1, \ldots, x_n), y^{(n)} \in \mathcal{W}^{(n)}.$ $\mathcal{M}^{(n)} \models \forall_{f(H)} G[\tilde{x}] \iff_{df} \\ \iff_{df} \mathcal{M}^{(n)} \models G \land \bigwedge_{\{k_1, \dots, k_m\} \subseteq \{1 \dots n\}} \Diamond_1 \dots \Diamond_n (\Diamond_{k_1} H \land \Diamond_{k_1} \neg H) \land \dots \\ \land \land \land \neg H) \rightarrow \Box_{k_1}$

$$\dots \wedge \Diamond_1 \dots \Diamond_n (\Diamond_{k_m} H \wedge \Diamond_{k_m} \neg H) \to \Box_{k_1} \dots \Box_{k_m} G$$

The acceptance relation is generalized to models, frames and classes of frames in the usual way. Then one explains the causal system C_L correlated with the modal logic L as $C_L =_{df} \bigcap_{n \in \omega} \{H \in FOR_f; \mathcal{K}_L^{(n)} \models H\}.$

In this way one produces a large manifold of causal systems. Jaśkowski's original calculus is among them: Q_f equals \mathcal{C}_{S5} . And what is more, all constructions of nonclassical logics semantically based on Kripke-frames (such as intuitionistic and tense logic or inconsistency tolerant calculi and first-order versions thereof) can be used to modify the above causal-logical approach. This gives a large number of different calculi. And consequently, any definition of a causal connective leads to an equally large variety of causal functors, since the specific properties of these functors differ from one system to the other. It obviously increases the chance of obtaining an appropriate formalization of the causal nexus.

The INUS Condition

Following ideas particularly of John Stuart Mill, some philosophers identify causes with conditions. The clear advantage consists in the well-formedness of our intuitions about conditions: we have sufficient, necessary, and sufficient and necessary conditions with established rules of application of these notions. Unfortunately, as John Mackie is not the only one to have mentioned, these notions are too strong in general. Sometimes we apply the term "cause" to conditions which are obviously neither sufficient, nor necessary for their supposed effects, let alone both of them together. In addition, sufficient and necessary conditions themselves — though sometimes regarded as basic — are analyzable in different manners. On a token level, with respect to singular events rather then to types of events, a counterfactual approach seems to be suitable. Against this approach, the following ideas of Mackie (cf. [16], though in this respect it differs from his [15]) are based on a regularity interpretation:

Definition (Necessary and Sufficient Condition)

Let X and Y stand for type of situation or event. Then, 'X is a necessary condition for Y' means that whenever an event of type Y occurs, an event of type X also occurs, and 'X is a sufficient condition for Y' means that whenever an event of type X occurs, so does an event of type Y.

This definition shows that — contrary to some statements — analysis in terms of necessary and sufficient causes does not require a modal garb. The first point of criticism still remains open: necessary and/or sufficient conditions are too strong to express causality. This is exactly the point at which Mackie introduces his ingenious explication of causation, by forming the notion of an insufficient but **n**on–redundant part of an **u**nnecessary but sufficient condition — an *INUS condition*. The intuition behind the INUS condition is easy to grasp; given a certain causal background (in the following: F) the cause is necessary for the effect among the given collection of conditions, but there could have been other collections of circumstances that produce the same effect. Only the totality of all possible collections which are sufficient for the effect is a necessary and sufficient condition. This is expressed by the definition:

Definition (INUS Condition)

A is an INUS condition of P in F if and only if, for some X and for some Y all F(AXorY) are P, and all FP are (AXorY).

Following Mackie's way of using disjunctive normal forms and extending it somewhat one can understand any elementary formulas (INUS conditions) as possible causes, elementary formulas describing the actual situation ("existing" INUS conditions) as causes:

Definition (Cause)

Let x_1, \ldots, x_n be elementary formulas, expressing simple conditions, and X a formula in disjunctive normal form whose elementary conjunctions contain x_1, \ldots, x_n . Let X — under F — express a necessary and sufficient condition of P. Then, all of the x_1, \ldots, x_n are possible causes of P, and all x_1, \ldots, x_n belonging to an elementary conjunction expressing an existing condition, are causes of P.

At first sight INUS conditions solve the frame problem for the causal nexus, which is particularly important for regularity theories. The question of which parts of the world one should include in the *ceteris paribus* clause is answered by reference to the remaining part of the elementary conjunction. Even better, any of the *ceteris paribus* conditions is an INUS condition and therefore able to play the role of a cause, in which case the "original" cause joins the *ceteris paribus* conditions. Nevertheless, there is no possibility of finding the *ceteris paribus* conditions or the other collections of INUS conditions that form the necessary and sufficient condition by logical means.

Recognizing and testing INUS conditions lies beyond the scope of logic. If there were a clearly defined way of carrying out these procedures and fixing the results it should be possible to introduce such heuristic aspects to the transparent and well investigated structure that disjunctive normal forms are. The remaining part of the section describes an attempt to bridge the gap between classical philosophical investigations and the implementation of causal operators in Artificial Intelligence using INUS conditions.

Following ideas of Mill — the method of differences (cf. below) — and Mackie — the INUS conditions — a *causal test strategy* was developed and implemented in PROLOG by Michael May (cf. [17]). It aims to show how it is possible to find and

to verify INUS conditions based on already known ones. Suppose, some "Mackie– structure" $hx_1 \lor x_2$ is assumed to be necessary and sufficient for an effect, where h are known INUS conditions, x_1 are the (unknown) remaining conditions making the elementary conjunction sufficient, and x_2 is the remaining part of the disjunctive normal form. May proposes to test new hypotheses and to fix the results in the form of a 4-tupel [Z1, Z2, Z3, Z4], summed up a test table as follows (here, *B* is tested against *A*):

(T0)	B	$\neg B$
A	Z1	Z2
$\neg A$	Z3	Z4

Each of the Zi is either 1 or 0, dependent on whether the effect occurs or not. A 4-tupel [1, 0, 0, 0] would show that B can be (conjunctively) added to A as an INUS condition: B is part of a minimal sufficient condition.

Although May later weakens some of the initial conditions of his analysis, the more elaborated system depends on the following assumptions:

- i. there is an (incomplete) causal hypothesis,
- ii. there is a test situation,
- iii. not explicitly mentioned relevant causal conditions are during a test either always present, or always absent, and
- iv. no different sufficient conditions contain the same INUS condition.

Formally, the last condition restricts disjunctive normal forms to formulas in which elementary conjunctions do not share any common elementary formula. Intuitively, the restriction is much to strong: it requires that all causes are causes only in one setting of conditions, and in no other.

Let K be the condition under test, and V (where it occurs) be that INUS condition against which K is tested. Let h, x_i be as mentioned above, then the following are the *rules of causal deduction* ([17], p. 70):

(T1)	$[1, 0, 0, 0], K, hx_1 \lor x_2$	$\longrightarrow hKx_1 \lor x_2$	
(T2)	$[0, 1, 0, 0], K, hx_1 \lor x_2$	$\longrightarrow h \neg K x_1 \lor x_2$	
(T3)	$[1, 1, 1, 0], K, hx_1 \lor x_2$	$\longrightarrow hx_1 \lor x_2 \lor Kx_3$	
(T4)	$[1, 1, 0, 1], K, hx_1 \lor x_2$	$\longrightarrow hx_1 \lor x_2 \lor \neg Kx_3$	
(T6)	$[1, 0, 1, 0], K, hx_1 \lor x_2$	$\longrightarrow hx_1 \lor x_2 \lor Kx_3$	
(T7)	$[0, 1, 0, 1], K, hx_1 \lor x_2$	$\longrightarrow hx_1 \lor x_2 \lor \neg Kx_3$	
(T8)	$[1, 0, 0, 1], V, K, hVx_1 \lor x_2$	$\longrightarrow hVKx_1 \lor x_2 \lor \neg K \neg Vx_3$	
(T9)	$[0, 1, 0, 1], V, K, hVx_1 \lor x_2$	$\longrightarrow hV \neg Kx_1 \lor x_2 \lor K \neg Vx_3$	
(T10)	$[1, 0, 1, 1], V, hVx_1 \lor x_2$	$\longrightarrow hVx_1 \lor x_2 \lor \neg Vx_3$	
(T11)	$[0, 1, 1, 1], V, hVx_1 \lor x_2$	$\longrightarrow hVx_1 \lor x_2 \lor \neg Vx_3$	
(T12)	$[0, 0, 1, 0], V, K, hVx_1 \lor x_2$	$\longrightarrow hVx_1 \lor x_2 \lor \neg VKx_3$	
(T13)	$[0, 0, 0, 1], V, K, hVx_1 \lor x_2$	$\longrightarrow hVx_1 \lor x_2 \lor \neg V \neg Kx_3$	
(T14)	$[0, 0, 1, 1], V, hVx_1 \lor x_2$	$\longrightarrow hVx_1 \lor x_2 \lor \neg Vx_3$	
May's aposition combination of the method of differences with the INUS			

May's specific combination of the method of differences with the INUS-condition approach allows for solving important epistemic problems of causality: of finding causal

relevant conditions, of recognizing minimal sufficient conditions, of recognizing interference factors, of recognizing irrelevant factors for minimal sufficient conditions, and of recognizing overdetermination and common causes. However, it depends not only on the assumptions mentioned above, but also on a regularity approach and on the assumption of type causation.

The Branching Time Approach

There are other attempts based on the notion of time. It has been suggested that a basic notion in defining causality is that of *branching time*. Properties of the causal nexus are then derived from the presumed ontologically open structure of the future.

The appropriate formal tools were first made available in the 1950's by Arthur N. Prior. Leaning on a profound knowledge of history of logic, he created a new logical discipline closely related to modal logic: tense logic. Saul Kripke suggested to him the idea of branching time. Prior incorporated this tree structure into the concept of time itself and was led to some philosophical reflections:

"Genuine determinism would be the belief that there is only one possible future, and to express this you really do need to go beyond K_t [the minimal tense logical calculus] and add a postulate for nonbranching of the future." ([19], 329)

Prior's attempt to create a formal theory of branching time had many followers. Storr McCall, for instance, claims that the passing of time is equivalent to a loss of possibilities, *i.e.* our understanding of this process reflects time as a branching system. (cf. [18]) Yet this view is not generally accepted. As Nicholas Rescher puts it, branching is *in* time, but there is no branching *of* time — time itself does not branch. ([21], 73 ff.)

A recent attempt is due to Nuel Belnap (see [1]) who uses the concept of branching space–time — "a simple blend of relativity and indeterminism" as he calls it ([1], p. 385). Gambling with technically rather uncomplicated tools — *i.e.* with point events ordered by a branching (causal) order — Belnap is quickly involved in a discussion of advanced topics in causal analysis in physics. His conception has a strong impact on philosophical issues such as indeterminism, the actuality of the future, and the status of assertions about future events whose occurrence is already settled (in contrast to the status of those statements about merely predicted facts).

In a recently written paper, Belnap and Green argue

"that one can make sense of an indeterministic, branching structure for our world without postulating an actual future as distinguished among the possibilities" ([2], p. 3)

The authors thus reject the common belief in the existence of a *Thin Red Line* (that represents the course along which history will develop) in favour of an open future.

In a similar framework of branching histories von Kutschera pursues his analysis of causation ([9]). The difference between him and the above authors is that von Kutschera does not take any notion of causation as primitive, but tries to define the concept of a cause from logically more basic notions. By a *cause* he understands

"an event which was not sure to have occurred and whose occurrence first guaranteed that of the effect. ... The effect, therefore, is conceived of as a necessary consequence of an event which in turn didn't occur necessarily. The concept of necessity employed is not a logical or nomological one. Necessary is rather what is the case no matter what turn the future history of the world will take." ([9], p. 563)

In his analysis it turns out that there are no causal chains at all. His basic notion is that of a tree–universe.

Definition (Tree-Universe)

A tree-universe is a pair $\mathcal{U} = \langle I, R \rangle$ such that

- 1. I is a set of world-states
- 2. *R* is a binary relation of immediate succession on *I* such that for all $i, j \in I$ we have
 - 1. the set of initial states I_0 is explained as $\{j; \neg \exists i : iRj\} \neq \emptyset$;
 - 2. $\forall i \exists j (j \in I_0 \land j R^{\geq 0})$, where $R^{\geq 0}$ is the ancestral of R;
 - 3. $iRj \wedge kRj \rightarrow i = k$;
 - 4. for all natural numbers m and n with m < n: $\exists i, k(i \in I_0 \land iR^nk) \rightarrow \forall i, k(i \in I_0 \land iR^mk \rightarrow \exists j : kRj).$

The concept of the set T of time points t, t', \ldots and of the set of worlds can then be defined. For each world state i there is an uniquely defined natural number z(i) representing its distance from some initial state $j \in I_0$. Then one has $T =_{df} \{n; \exists i(z(i) = n)\}$ and $W =_{df} \{w \in I^T; \forall t(w(t)Rw(t+1) \land w(0) \in I_0)\}$. Subsequently, he deals with the notion of event, defined as sets of segments of worlds. Events occur at most once in every world, and they have a well–defined beginning and end, coded by some time interval $\tau = [t_1, t_2]$.

Definition (Event)

An event is a set E of segments of worlds such that

- 1. $\forall w_{\tau}, w_{\tau'} \in E \colon \tau = \tau'$;
- 2. $\forall w_{\tau}, w'_{\tau'} \in E \colon w_{\tau} \cap w'_{\tau} \neq \emptyset \to \tau = \tau'$

von Kutschera understands a cause as something whose occurrence for the first time guarantees the occurrence of the effect. In order to state this precisely some further abbreviations are helpful. The state of affairs that E occurs is formally expressed by $E^0 =_{df} \{w; \exists \tau : w_\tau \in W\}$. Furthermore, let $W^{w(t)}$ be the set of all worlds passing through w(t). E is determined in w and t [symb.: D(E, w, t)] iff $W^{w(t)} \subset E^0$ and E is determined in w from its beginning [symb.: Db(E, w)] iff there is an time interval τ such that $w_\tau \in E$ and D(E, w, t).

Definition (Cause)

The event E causes the event E' in the world w [symb.: K(w, E, E')] $\iff_{df} \exists \tau (w_{\tau} \in E \land$

$$\wedge \forall w', \tau'(w' \in W^{w(\tau_1)} \land w'_{\tau'} \in E \to DB(E', w') \land \neg D(E', w', \tau'_1))) \,.$$

von Kutschera goes on to elaborate his approach in which appears to be one of the most interesting and promising recent attempts of formal causal analysis.

The Probabilistic Analysis

Serious philosophical questions and objections are often raised against probabilistic analysis, such as those concerning the understanding of probability, the interpretation of (the exclusion of) border cases, the concrete choice of the base of conditionalization, and others. But it is unfair to burden *probability theory of causation* with all these problems; rather a lot of them belong either to the probability theory in general, or to the causal theory in general. The probabilistic analysis obviously satisfies many of the intuitions connected with causality: that the occurrence of the cause makes it more likely for the effect to occur than without this occurrence, that causes open new (possible) ways, that one could have expected the effect by knowing the cause in advance. The probabilistic framework — which is quite common in several sciences — allows us to express the main idea that a cause is a possible event that raises the probability of its effect. This kind of analysis comes in many forms.

In order to understand the basic ideas underlying the probabilistic approach it is useful first to have a look at one of the earliest attempts to link probability theory with philosophical analysis of causation. Hans Reichenbach's initial idea (cf. [20]) consists in two parts: a cause produces its effect, while an effect records its cause; and in order to use this distinction in his analysis, he develops his famous mark method. According to Reichenbach a mark is the result of an intervention by means of an irreversible process. It is possible to define a relation "causally between" without further reference to other causal concepts (P(A, B) means the probability of the sequence "A and then B"):

Definition (Causally Between)

An event A_2 is causally between the events A_1 and A_3 if the following relations hold:

- 1. $1 > P(A_2, A_3) > P(A_1, A_3) > P(A_3) > 0$,
- 2. $1 > P(A_2, A_1) > P(A_3, A_1) > P(A_1) > 0$,
- 3. $P(A_1, A_2, A_3) = P(A_2, A_3).$

The third condition formulates the important idea, that "nearer" causes screen off the efficiency of distant causes. The aim of the construction is to define causal relevance, which is the philosophical backbone of Reichenbach's conception: If a mark made in an event A_i shows in an event A_k , then A_i is causally relevant to A_k . The following assumptions are made in order to link the relation defined above with the marking process:

Assumption α . If a mark made in A_i shows in A_k , then $P(A_i, A_k) > P(A_k)$.

Let A' be the event resulting when the mark is added to A:

Assumption β . If a mark is made in A_i , then either

 $P(A'_{i}, A'_{k}) = P(A_{i}, A_{k}),$ or $P(A'_{i}, A_{k}) = P(A_{i}, A_{k}).$

- **Assumption** γ . If A_2 screens off A_1 from A_3 , and if a mark made in A_1 shows in A_3 , then it also shows in A_2 .
- Assumption δ . If a set $A_2^{(1)} \dots A_2^{(n)}$ screens off A_1 from A_3 , and if a mark made in A_1 shows in A_3 , then it also shows in at least one of the events $A_2^{(1)} \dots A_2^{(n)}$.

The assumptions express Reichenbach's conviction in action by contact. A final definition fixes the central point of probabilistic analysis of causation:

Definition (Causal Relevance)

An event A_1 is causally relevant to a later event A_3 if $P(A_1, A_3) > P(A_3)$.

Quite obviously, Reichenbach anticipated a lot of ideas that were later incorporated in more sophisticated treatments.

One of the more elaborated probabilistic accounts of causation is due to Patrick Suppes, who tried to build up a formal framework on the basis of some of Reichenbach's ideas. The system is explicated in a series of definitions, taken from his [25]:

Definition (Prima Facie Cause)

The event $B_{t'}$ is a prima facie cause of the event A_t if and only if

i. t' < t,

ii. $P(B_{t'}) > 0$,

iii. $P(A_t|B_{t'}) > P(A_t)$.

Suppes now faces the problem of *spurious causes*, that is events which appear to be causes, but which are not quite what they appear to be. In order to grasp the really *effective* causes, he has to exclude co–occurrent events not causally connected, but nevertheless related according to the definition above. He first defines spurious causes in one sense based on other effective events, and then moves on to define them in another sense based on the occurrence of events of a certain type. The following two definitions differ in their existential requirement: one is concerned with singular events and the other with event types.

Definition (Spurious Cause I)

An event $B_{t'}$ is a spurious cause in sense one of A_t if and only if $B_{t'}$ is a prima facie cause of A_t and there is a t'' < t' and an event $C_{t''}$ such that

i. $P(B_{t'}C_{t''}) > 0$,

ii. $P(A_t|B_{t'}C_{t''}) = P(A_t|Ct''),$

iii. $P(A_t|B_{t'}C_{t''}) \ge P(A_t|B_{t'}).$

As Suppes claims this definition makes a prima facie cause spurious if there exists an earlier event that eliminates the effectiveness of the cause when that event occurs.

According to him, condition (iii) imposes a rather strong constraint on this earlier event. All examples he looks at suggest an alternative approach in which the third condition is dropped and a partition of the past before the spurious cause is introduced so that for every element in the partition, condition (i) and (ii) hold. Intuitively, this corresponds to the stated requirement — that if one can observe an event of a certain kind, a type of earlier event, then knowledge of the spurious cause is predictively uninformative:

Definition (Spurious Cause II)

An event $B_{t'}$ is a spurious cause of A_t in the second sense if and only if $B_{t'}$ is a prima facie cause of A_t and there is a t'' < t' and a partition $\pi_{t''}$ such that for all elements $C_{t''}$ of $\pi_{t''}$

- *i.* $P(B_{t'}C_{t''}) > 0$,
- *ii.* $P(A_t|B_{t'}C_{t''}) = P(A_t|C_{t''}).$

Suppes' construction is closely related to everyday intuitions of causality. They allow for definitions expressing the straightforward understanding of how causal notions work, as the following two examples show:

Definition (Direct Cause)

An event $B_{t'}$ is a direct cause of A_t if and only if $B_{t'}$ is a prima facie cause of A_t and there is no t'' and no partition $\pi_{t''}$ such that for every $C_{t''}$ in $\pi_{t''}$

- *i.* t' < t'' < t,
- *ii.* $P(B_{t'}C_{t''}) > 0$,
- *iii.* $P(A_t | C_{t''} B_{t'}) = P(A_t | C_{t''}).$

"Direct causes do not have mediate effective events"; and

Definition (Supplementary Causes)

Events $B_{t'}$ and $C_{t''}$ are supplementary causes of A_t if and only if

- *i.* $B_{t'}$ is a prima facie cause of A_t ,
- *ii.* $C_{t''}$ is a prima facie cause of A_t ,
- *iii.* $P(B_{t'}C_{t''}) > 0$,
- *iv.* $P(A_t|B_{t'}C_{t''}) > \max(P(A_t|B_{t'}), P(A_t|C_{t''})).$

"Supplementary causes are together at least as effective as each of them".

A different formal approach to probabilistic causation can be found in the work of Wolfgang Spohn. He uses at least two relativizations which clarify his understanding of causation: first, causation is world dependent — there may well be two worlds that differ in prescribing a causal relation to two existing events; and second, causation is context dependent in a world — in some way one should take the history of both the cause and the effect into consideration. Although Suppes recognizes the need for relativization (in fact, that is the reason for introducing spurious causes), he does not

give us a formal access to the matter. Spohn does, but this at the price of a much more complicated structure.

Let *I* be a finite non-empty set of variables with a weak ordering \leq (representing time), every $i \in I$ being associated with a finite set Ω_i of at least two values. The cross product Ω of all the Ω_i is the set of all functions (possible worlds) ω such, that for each $i \in I$ it holds that $\omega_i \in \Omega_i$. Let $\{ < j \}$ denote the past of $j \in I$ and $\{ < j - K \}$ the past of $j \in I$ except $K \subseteq I$ ($\{k \in I | k < j\}$ and $\{ < j \} - K$, respectively). For each $\omega \in \Omega$ and $J \subseteq I$ let ${}^{\omega}J$ be $\{v \in \Omega | v(i) = \omega(i)$ for all $i \in J\}$, and the subset A of Ω (being a state of affairs) is a J-state iff, for all v and ω agreeing on J, $v \in A$ iff $\omega \in A$.

Now it is possible to express the idea that every cause raises the probability of its effect under all circumstances given prior to the effect (but excluding the cause itself):

Definition (Direct Cause)

Let A be an *i*-state, B a *j*-state, i < j, and $\omega \in A \cap B$. Then A is a direct cause of B in ω iff $P(B|A \cap \omega \{ < j - i \}) > P(B|\overline{A} \cap \omega \{ < j - i \})$.

In a series of definitions Spohn clarifies what *circumstances of a direct cause* could be, definitions which range from mere temporal priority to the idea that circumstances include the other causes and counter–causes of an effect. Causation, as including direct and indirect causation, should be found somewhere between direct causation by itself and its transitive closure. Within this framework Spohn — after a discussion of other variants and arguments against transitivity — opts for the "upper bound":

Definition (Causation)

Causation Relation is the transitive closure of direct causation.

By introducing the circumstances explicitly into the analysis, Spohn is able to overcome some of the most common counterexamples to the probabilistic theory of causation. However, he has to argue against the objection that there are too many facts in his conditionalization basis.

The Axiomatic Approach

One of the first contributions to causal logic was Greniewski's paper [5]. Here, the author lays out his program of causal analysis within mathematical logic.

- 1. Produce a definition of the concept "A is the cause of B" (by means of axiomatically characterized notions of "hyperspace" and "temporal order") that meets all requirements of modern logic and which describes as perfectly as possible the usual meaning of "A causes B".
- 2. Construct a deductive theory of causality based on the above definition and on possibly further definitions of causal terms.
- 3. Formulate the postulates of determinism and indeterminism and investigate their relations to the postulates of the causal theory.
- 4. Check the applications of this causal theory (are there any, and if there is, which ones?)

The strategy of this early work on causal logic looks amazingly up-to-date. The properties of the causal connective result from the postulated structure of the world. In such cases one faces the danger of tieing the causal relation too closely to the temporal succession. The approach avoids the usual difficulty of axiomatic attempts to a causal analysis, however. One assembles the axiomatics from the formal counterparts of the intuitive properties of the concept considered. In the case of the causal nexus there is a considerable shortage in commonly accepted properties. Therefore good candidates for axioms of the formal calculus are not at hand.

Nevertheless, there are other attempts in axiomatic causal logic, Burks ([3]) is a very early one as we have seen. Other authors, starting with axiomatic considerations, pass on to semantical investigations at the first opportunity ([4], [26]). It is of course possible to search for axiomatizations of semantically explained classes of tautologies. Although mathematically interesting, this problem merits no special interest from a causal–logical point of view.

Philosophical Interpretations

Indeed, the formal methods used in the logical analysis of causation do not predetermine the features of the causal nexus in the world. What logic can do is to produce propositions of how to speak consistently about reality. It is up to the philosopher to decide which of these possible terminologies is the most appropriate one, which one fits the world best.

A classical approach to the philosophical treatment of causation was developed by David Hume. On the basis of his conception of sense impressions and simple ideas, he thought that there was nothing in our experience which provided us with a simple idea of the causal nexus as a necessary connection between events. Hume therefore claimed that since we have this idea of a necessary connection, of a causal power, it must stem merely from our habits of thinking. What we really have — according to Hume — are sense impressions (of events) succeeding each other time after time. The idea of causality arises as a result of the regularity of the temporal succession between events of a similar kind. As a result there is no causal dependency different from temporal succession.

In its most developed form the Humean account involved the notion of a law in the sense that causal beliefs will be valid if (and only if) the relationship between two events can be subsumed under a general law expressing a regularity. This idea gave raise to a widely accepted theory of causal explanation which usually takes the form of the famous D–N–Model of Hempel and Oppenheim: to explain means to deduce a sentence from two kinds of sentences. Thus, the explanans contains at least one general sentence (the law) and one sentence describing the boundary conditions and the actual observation, and the explanandum follows by deductive inferences. But it is well known that this attempt faces the notorious frame problem concerning the difficulties with a complete description of the boundary conditions; a problem which appears not only in explanation theory but also in recent research in Artificial Intelligence.

An attempt to clarify the notion of a cause both with the help of the D–N–Model of explanation and with the help of the notion of a causal law was made by Wolfgang

Stegmüller in his [24]. According to him the problem of causality involves the topics of causal law, scientific explanation, causal explanation, causes, and the general principle of causation. These topics are related as shown in figure 1 on page 17, where the arrows depict dependency.

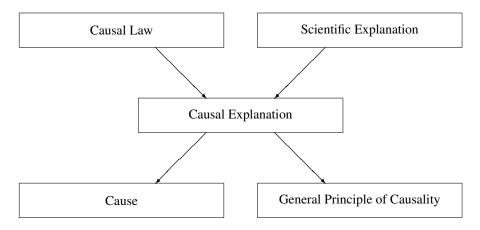


Figure 1: The Problem of Causality

Clearly, we now have first to define "scientific explanation" and "causal law". On this approach, a scientific explanation is an explanation via the D–N–Model. It is very hard to define a causal law, and each of the following characteristics connected with its own difficulties and problems: A causal law is a quantitative, deterministic, micro-, succession-, near-distance law, expressible by continuous mathematical functions, which refers to a homogeneous, isotropic spatio–temporal continuum. Thus, according to Stegmüller, causal laws are very specific, tricky laws of nature. A causal explanation is, then, defined as a scientific explanation in which the laws are causal laws. This opens the possibility of defining a cause as in the next definition:

Definition (Cause)

Suppose, the event E has an adequate causal explanation. Then the cause of E is the totality of antecedent conditions figuring in the explanans of E.

One has to agree with Stegmüller who doubts whether the above defined notion of a cause has any theoretical or practical relevance.

The Humean account has been extremely influential over the centuries and has been very much favored by empirically minded philosophers. Despite this, there have always been alternative theories, although they never gained general support. Eventually Hume's phenomenalistic epistemology became more and more difficult to defend and the foundation of his causal theory was accordingly undermined. Another development which brought Hume's regularity account into discredit was the growth of intensional logic in the first half of our century. Likewise the construction of the possible worlds semantics and the subsequent rapid development of modal and relevant logic in the fifties meant a breakthrough in our understanding of counterfactuals. This prepared

the ground for a new philosophical approach to causality where causal dependency between external events is not seen as an accidental feature of our thinking. With the new logical tools it has become obvious since the sixties that one should try to analyze causal dependency in terms of counterfactuals by the means of possible worlds semantics.

Part of the advantage of the counterfactual account is seen in the fact that it explicitly ascribes a modal force to a cause which is missed in the regularity analysis. This modal force is reflected in our talk about sufficiency and necessity: not only the occurrence of the cause produces the effect, but also the lack of the cause necessitates the lack of the effect. It is a hope of those who use such an analysis to be able to give a full account of causation, including causal dependency as well as causal priority. Other people require more than counterfactual dependence in order to grasp the meaning of causation. They often rely on ideas of temporal order or on the notion of propensity instead.

It is obvious that causation and time are closely associated; the question is which of them, if any, is ontologically more basic. All possible opinions concerning are to be found on this matter. According to Hume and his successors the only way a regularity theory can provide a causal ordering relation is by building upon temporal orientation. This excludes the conceptual possibility of backward causation and even of simultaneous causation which some people want to defend. Thus, a well–known objection to the idea of defining causal priority by mere temporal priority is that this notion reduces *propter hoc* to *post hoc*. The epistemological ground for claiming that the causal orientation has to be defined in terms of temporal succession would be the suggestion that we directly perceive the temporal order of events. Those people who argue against this opinion believe that the temporal order is supervenient on causal processes or, given that backward causal processes exist, that it may merely be conventionally connected with the most dominant processes.

Another alternative to the counterfactual analysis has focussed on probabilities and related topics. The probabilistic analysis of causation is motivated by objections against the basic abstractions of regularity and of the counterfactual theory. If a single causal statement is only a mere instance of a causal law, or something which is completely governed by laws, we would certainly leave us with very few true singular causal statements. There seems nevertheless to be too many exceptions of the so–called laws (and that is one of the aspects of the frame problem). However, if we deal with a counterfactual analysis of singular causal statements, and the cause event and the effect event occur, we have to know how the world would look if the cause had not occurred. But how can we? Certainly not with complete precision. So, the proposal is instead that we should consider probabilistic laws in the case of regularity theory, or — more commonly — that we should in the second case consider not a fictitious world but rather a conditional probability.

With this kind of approach the problem of determinism and indeterminism becomes very vivid. On the one hand, the classical view was that the world is completely deterministic in the sense that every event has a cause and this cause is sufficient for the effect. Very often this *ontological* sense of the term is confused with the *epistemical* sense of predictability. But only for a Laplacian Demon will predictability and the ontological conception of determinism coincide. On the other hand, since the intro-

duction of Quantum Mechanics there is general agreement that the world might not be deterministic. In the realm of the quantum world objects behave indeterministically in the sense that we cannot ascribe a sufficient cause to every incident. Nevertheless, the possibility of prediction is still on hand as one has been able to formulate statistical laws. This has given rise to a natural way of interpreting these statistical laws as expressing objective chance or *propensities*.

Others have seen statistical laws and probabilities as useful when we want to describe complex systems where our knowledge cannot cope with all the details. Either these details are about the behavior of a huge number of objects, or they are concerned with the exact values of the initial conditions. In the first case the probabilities are then usually interpreted as an expression of the lack of our knowledge or seen as reflecting patterns of frequencies, but in the second case the possible interpretations of probabilities are much more open. In addition to the first case one can face principal uncertainties in dealing with probabilities. Consider causal connections in dissipative dynamical systems which reveal chaotic behavior (if there are any such systems). Then the differentiation between "deterministic" (as the idea of a system being perfectly describable by a set of differential equations) and "probabilistic" is no longer a sharp one. In such a case the system develops deterministically, but predictions can be stated with some degree of probability only. This is due to a specific feature of these systems: in the long run no precise calculation of future states of the system is possible. All we can know is based on probabilities. For instance, the average temperature of next April cannot be established on the base of today's meteorological measurements, but it is still possible to get some reasonable data based on statistical material covering a large sample of average temperatures of the period concerned in previous years. Therefore one might put it not as "either probabilistic or deterministic" but rather as "probabilistic and yet deterministic".

Apart for the various possible interpretations of probabilities — whose existence by themselves casts a shadow over the idea of reducing causal statements to probability statements as our grasp of the former seems at least as firm as that of the latter — the probabilistic approach suffers from other well–known problems. One is that we have no difficulty in talking about events that probabilistically seem to be negatively relevant for their effects (but which *are*, nevertheless, genuine causes), whereas probabilistic causation is so defined that causes are positively relevant for their effects. Another difficult problem to handle is the notion of spurious causes. If one uses only single probability functions it is impossible to distinguish between a causal chain and a fork when the probabilities involved are identical. These kinds of problems provide some philosophers with a feeling that it is impossible to reduce causal statements to probability statements without any residue.

But there are still other attempts to define causality. Closely associated with the notion of causation is the conception of human agents. We see ourselves as capable of producing results and changes in the world by performing actions. It therefore seems natural to suggest that our ideas of causes are derived from the immediate experience of our power of manipulating things and bringing about what we intend to happen. Given this train of thought one can understand, for example, C.H. von Wright's idea of defining causation by reference to the interference of agents.

Definition (Cause and Effect)

P is a cause relative to Q, and Q an effect relative to P, if and only if by doing P we could bring about Q, or by suppressing P we would remove Q or prevent it from happening.

Von Wright then combines his definition with the idea of the world as consisting of branching systems of states. This allows him to talk about different possible courses of the world consequent upon different patterns of action that might be chosen by the agent.

However, a conspicuous objection to any attempt at defining the causal nexus in terms of what we are capable of doing is that it becomes impossible to distinguish between bringing about Q by doing P in the sense that P causes Q and bringing about Q by doing P in the sense that P entails Q. This shortcoming may be taken as an indication that it is the reverse case which is more likely; namely, that the concept of causation is necessary for an understanding of what it means to *bring about Q by doing* P. Actions are, moreover, commonly described in an intentional language, where the intentional manner of description refers to goals at which people are aiming, whereas causal descriptions leave out any kind of teleological conception of the action as a whole. As empirical sciences of man have developed, there has been an increasing interest in getting rid of teleological considerations. Instead of trying to define causation in terms of actions many scientists and philosophers have over the years attempted to reduce statements about people's actions to causal statements. But the truth may very well be that neither the concept of action nor the concept of causation can be reduced to one another.

A natural reaction to the various drawbacks which seem haunt every attempt to analyze causation in terms of other notions like regularities, counterfactuals, branching times, agents, or probabilities would be to argue that the meaning of causal sentences cannot be completely given by the meaning of any other kinds of sentences. These other notions are of course closely connected with causality but the explication of any of them has its own independent problems. One should therefore consider causality and any of the other notions as separated from one another although they are at the same time related to one another in such a way that a complete understanding of one of them has to take the others into account. That is why sentences reflecting any of these other notions generally can be used to report causal happenings.

Formal approaches can be considered as attempts at giving an explication of the causal notion and of the reasoning based on it. Another thing which philosophers have discussed over the centuries is what are the features in our epistemic situation that gives rise to causal reasoning, and how does one find out whether there are causal bonds in nature. There must be some methodological rules to follow assuring that our causal reasoning sometimes is applicable to what we experience. Already in the seventeenth century Francis Bacon set out some methods for establishing causality. One should draw up three tables: one containing the positive instances of the phenomenon to be explained in all its variation; another covering the negative instances that were similar to the positive instances but without exhibiting the phenomenon in question; and a third consisting of a list of positive instances in which the instances are ordered according to the intensity of the phenomenon regarded as the effect. From the tables the cause

could then be found by seeking a conjunction of properties such that the conjection can be obtained among all positive instances, but not among every negative instances, and it increases its intensity at the same time as that of the phenomenon.

Another important historical contribution to the development of causal methods was made by John Stuart Mill who, like Bacon, suggested three possible ways for inferring causes. The first one is the method of agreement where one infers that the cause is those properties common to all instances of an effect. The second is the method of difference where one infers that causes are those properties present to all instances of an effect but absent from all instances taken not to be an effect. Finally, the third one is the method of concomitant variation where one infers that the cause is the combination of properties that increases in intensity as the effect increases in intensity but decreases when that of the effect does. Basically, it is these three methods that make up the foundation for a modern statistical analysis of correlations between events.

What kind of inclinations one harbours towards the various formal approaches to an analysis of causation hinges very much on one's general philosophical outlook. If someone has a strong empirical or non–realist attitude to semantics and epistemology, he or she would be liable to look positively at those approaches that contain as few elements as possible lying outside the range of our experience, or as many elements that can easily be interpreted in accordance with the ideals of empiricism. Such approaches would include accounts based on regularity, branching time, and on some versions of the probability approach, namely those cases where probabilities are interpreted as a measure of the lack of knowledge or as expressions of observable frequencies. Others have more realist inclinations towards meaning and knowledge, and they will therefore seek to develop approaches open for a realist understanding. Those include accounts building on counterfactuals, other modal approaches, and at least one version of probabilities where these are seen as expressions of propensities or objective changes.

It is therefore obvious that which kind of analysis of the causal nexus is regarded as satisfactory is not something that is decidable simply on the grounds that the analysis is consistent with our ways of speaking about causal matters. The question of one's philosophical perspective on the world is a determining factor of which analysis one prefers. Which one is considered as conveying our intuitions most accurately depends very much on arguments drawn from other areas of philosophy. This explains why causal analysis is so challenging.

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