Carnap's construction of the world

The *Aufbau* and the emergence of logical empiricism

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To my mother and father
Elizabeth Catherine Fredricks Richardson
and
Edward Thompson Richardson

What does Carnap commit himself to in accepting a structuralist account of objectivity? What are the details of the two different projects that are meant to express the import of this structuralism? Why are there two such projects? Ultimately, considerations of these issues will lead to a general examination of a central tension among Carnap's notion of epistemology guiding the technical projects, the centrality of structure in those projects, and his understanding of logic as type theory. In the end, we will be in the position to see why Carnap gave up epistemology by the mid-1930s and became the type of philosopher of science he became.

Before indulging in high-level interpretation, however, it will be best to review some of the features of Carnap's structuralist program and its two expressions. These aspects of the *Aufbau* are less well known than they might be, and we will have to engage in a fair bit of exegesis. In order to fix ideas, let us turn to this project first. The reader's patience is begged. In return, the author agrees to avoid mixing the fairly low-level exegesis with too many preliminary high-level interpretative remarks. A number of philosophical points that might be made will, therefore, be held off until Chapter Eight, when our background trip through the neo-Kantians and Carnap's pre-*Aufbau* work is finished and the whole of the interpretative perspective is finally available.

The exegetical remarks on the epistemology of the *Aufbau* will form the next two chapters. In Chapter Two, the focus will be on a more detailed discussion of the two halves of Carnap's epistemological problem – the subjective and the objective. Then we will move to a discussion of some of the principal features of the logical procedures he exploits in the project. The primary topics will be his account of purely structural definite descriptions and the procedure of quasi analysis. Chapter Three presents a broad outline of the constitutional system that Carnap presents in the *Aufbau.* Here, again, the main order of business will be the presentation of the two projects for objectivity.

CHAPTER TWO

The problem of objectivity: An overview of Carnap's constitutional project

In this chapter we shall examine more closely the epistemological problem of the constitutional system of the *Der logische Aufbau der Welt*, as well as some of the technical means that Carnap proposes to use to solve this problem. Our first order of business will be to examine the import of the epistemological vocabulary used to motivate the projects of the work. This vocabulary emphasizes the subjective–objective distinction. It is incumbent upon us, then, to try to acquire a better sense of Carnap's use of these terms.[1] Most particularly, we must try to understand what is at stake in the claim that structure yields objectivity.

The main purpose of this chapter is to present Carnap's major motivating ideas as an introduction to his thought on objectivity. In Chapter Three, we shall examine his particular solutions to the problem of objectivity in the *Aufbau.* The point is to emphasize the crucial interpretative issues raised by the work on their own terms, while freeing ourselves from the need to cram their significance into ready-made philosophical pigeonholes. The puzzles that we will be left with at the end of these chapters and the question of how they arise within the philosophical context of Carnap's book will be our major concern in the remainder of the book.

1 "Objectivity" has recently reemerged as a central and vexed term in epistemology and metaphysics. For recent views on objectivity, see, e.g., Nagel (1979) and Putnam (1990). Within the history of science, the history of the concept of objectivity has recently become a topic of research. See, e.g., the papers from the symposium on objectivity published in *Social Studies of Science* 22 (1992), by Lorraine Daston, Ted Porter, and Peter Dear, respectively, and also Daston and Galison (1992).

KNOWLEDGE VERSUS EXPERIENCE: THE PROBLEM OF SUBJECTIVE ORIGINS

Let us recall the form of the epistemological problem that Carnap presents at the opening of his book: Knowledge, for any given agent, begins in the stream of experience of that individual. Individual experience is, however, the paradigm of the subjective; the qualitative particularities of any stream of experience are wholly private to the person whose experience it is. As such, the peculiarities of that experience cannot be communicated to any other agent. The problem is, then, to show how "it is still possible . . . to attain to an intersubjective, *objective world* that is conceptually comprehensible and, indeed, as one identical for all subjects" (§2). Carnap's solution to this problem is twofold. First, there are structural features in common among the individual streams of experience. Second, one can define the objects of knowledge wholly on the basis of such shared structural features of experience.

Carnap's most perspicuous statement of this structuralist view of objectivity, given in section 66, is worth quoting in full:

> The problem now reads: How is science to come to intersubjectively valid statements if all its objects are constituted from an individual subject, if therefore all statements of science have in the end only relations between "my" experiences as their object? Since the stream of experience of any person is different, how is even one sentence of science to be objective in this sense, i.e., valid for every person, if it proceeds from his individual stream of experience? The solution to this problem lies in that of course the *material* of the individual streams of experience are completely different, or rather not comparable at all, since the comparison of two sensations or feelings of different subjects in the sense of their immediate given quality is nonsensical; but certain *structural properties* agree for all streams of experience. Science must restrict itself to such structural properties, since it is to be objective. And it can restrict itself to structural properties, as we have seen earlier, since all objects of knowledge are not content but form and can be represented as structural entities.

From the point of view of the formal logic of the *Aufbau*, this means that all streams of experience have some recognizably similar logico-mathematical structure. In practice, the streams of experience are conceived as ordered by a single relation, termed *recollection of similarity* (Rs), which holds between two elementary experiences of any given subject when one such experience is similar to a recollected representation of the other. (An elementary experience of an agent is a complete cross-section of experience for that agent at a given time.) The recollec-

tion of similarity relation is sufficiently structurally similar across agents to allow the objects of science to be defined from its structure. This procedure yields objects common to and identical for all agents. Before looking more at intersubjectivity and structure, let us note a few consequences of the subjective starting point of epistemological enquiry.

Presenting the problem of epistemology as the movement from subjective experience to objective knowledge brings with it important consequences. First, it provides the reason for the choice of the autopsychologically based constitutional system. Only because we are interested in capturing the epistemological order within the constitutional system do we choose such a system. But, as noted in Chapter One, this decision is not based on the idea that only the given in experience is real, nor even that the deliverances of sensation are the certain starting point for knowledge. Rather, it is based on the scientific fact and epistemological presumption of all philosophical schools (§178) – that knowledge begins in experience – that guides this choice. Moreover, since primitive experience is subjective and private, epistemological honorific notions like 'certain' simply do not apply to it.

Correlative with this, the subjective starting point of the epistemological constitutional system requires an endorsement of methodological solipsism. Methodological solipsism for Carnap amounts to "an application of the form and method of solipsism" (§64). Methodological solipsism is not a commitment to the truth of any judgment about the sole or preeminent reality of some particular epistemic subject, however. Methodological solipsism is simply a commitment to a type of constitutional order as the order that captures epistemological relations. It is that order toward which all epistemological schools tend. "Solipsist" or "autopsychological" are, moreover, not terms that have any determinate content prior to the constitutional project itself. They are not, therefore, metaphysical terms that must be imposed from outside of the constitutional system in order to make sense of the project as a whole. Carnap writes (§65):

> "Methodological solipsism" [is] not meant as if it presupposed at the start a separation of the *"ipse,"* the "ego," from the other subjects, or as if one of the empirical subjects is set off and understood to be the epistemological subject. At the start, neither other subjects nor the ego can be discussed. Both are first constituted indeed simultaneously at a higher level. The choice of these expressions means merely that *after* the complete construction of the entire constitutional system, there are in hand differing domains that we call, in accordance

with usual designations, the domains of the physical, of the psychological (i.e., of the auto- and heteropsychological), and of the cultural [Geistigen].

Thus (perhaps confusingly, given subsequent usage), for Carnap "methodological solipsism" amounts to the same thing as "neutral monism."[2] Since the meaning of these motivating distinctions among objects occurs only within the system, the basis for any system is, at the start, neutral. Thus, he continues in section 65 to claim:

> The characterizations of the basic elements of our constitutional system as "autopsychological," i.e., as "psychological" and as "mine," first has a meaning when the domains of the nonpsychological (at first, the physical) and of "you" have been constituted. Then, however, it is thoroughly meaningful for distinguishing this system form from the other system forms with either a general psychological or a physical basis.

How, then, can we understand what it is we are trying to do in providing a constitutional system prior to actually doing it? In particular, how are we to understand the starting point of the epistemological constitutional system as "subjective"? That is, how are we to understand Carnap's argument that the epistemic order requires an autopsychological basis (§§54, 64)? Surely we are occupying a philosophical point of view outside any constitutional system in giving such arguments. Perhaps, therefore, we need a prior, metaphysical standpoint before we can understand the motivation for Carnap's epistemology.

This is precisely what Carnap seeks to deny. He argues that we are not employing metaphysical terms in such considerations. Rather, we are engaged in discussions in the as yet unreconstructed sciences. That is, in our example, the notions of 'experience' and 'autopsychological' are psychological, not metaphysical, notions. Thus, for instance, in his preliminary discussions of the lower constitutional levels (§§75–94), Carnap is happy to relate what is being done in the constitutions through discussing them in relation to cognitive facts within the as yet unreconstructed language of psychology.

Indeed, Carnap's account of the elementary experiences and the basic relation of recollection of similarity are wholly informed by Gestalt psychological findings about human experience. This is an empirical constraint that is built directly into the basis of his epistemological

2 On issues around solipsism and neutral monism in positivism and logical empiricism, see Hamilton (1990, 1992).

system. Psychology is where we must look if we want to know what human experience is like. Thus, for example, in defending the idea of the epistemic primacy of complete cross-sections of experience and, therefore, the choice of the elementary experiences as the basis elements, Carnap writes (§67): "Contemporary psychological research has confirmed more and more that in the various sense modalities, the total impression is epistemically primary, and that the so-called individual impressions, from which one is subsequently accustomed to saying the perception is 'composed,' are first achieved through abstraction therefrom." Carnap's desire to capture epistemic primacy and his endorsment of looking to the sciences for the indicators of primacy are, therefore, his sole reasons for demanding that the basis elements have this holistic structure.

Therefore, the findings of the empirical sciences, and especially psychology, are used within the epistemically ordered constitutional system from its very basis. Of course, these findings of psychology are themselves tentative and subject to revision. In this way, the constitutional system is itself subject to change (cf. §106), motivated wholly by changes in our best scientific knowledge in psychology and the other sciences.

Hence, the subjectivity of the starting point is motivated by empirical psychology, and our knowledge of empirical psychological theories will be clarified by the constitutional system motivated by its findings. The constitutional project can be seen, therefore, as a project of clarification of the basis of scientific knowledge from within. The concern with the connection of objective concepts in the sciences reflects and makes precise the partially unarticulated and intuitive procedure used by the scientists. It is for this reason that Carnap calls it a "rational reconstruction of the process of cognition" (§100).

STRUCTURE AND OBJECTIVITY

We now have a tolerably clear understanding of how Carnap means to delimit his epistemological starting point – how he conceives of the subjective. But what of the goal of objectivity? His claim that all scientific objects are (§66) "not content but form and can be represented as structural entities" requires clarification. What is motivating Carnap's views here? What, in the end, does it mean to be a structural entity?

Carnap, in effect, has two paradigm cases in mind. In an early discussion of objectivity in empirical science (§16) it is clear that physics exhibits a position as the preeminently objective empirical science: "In physics we easily notice this desubjectification, which has transformed almost all physical concepts into pure structure concepts." Furthermore, although Carnap is at pains to stress the objective status of other sciences, it is clear that the objective status of those sciences is derivative upon the objectivity of physics. Of the constitution of the physical world, Carnap writes (§136):

> The goal of this constitution consists in the introduction of a domain that is determined through mathematically expressible [fassbar] laws. The laws are to be mathematically expressible in order that with their help certain conditions can be *calculated* from others, which determine them. The necessity of the constitution of the physical world rests furthermore on the circumstance that only it, but not the perceptual world . . . permits a univocal, contradiction free intersubjectification (§§146–9).

However, there is another paradigm that informs the structure of the constitutional system and the account of objectivity it subserves. This is the constitutional system of the concepts of mathematics given by Whitehead and Russell in *Principia Mathematica* (§12). But Carnap's remarks about the relation between the logicist reduction of mathematics to logic and the constitution of the concepts of empirical science in the autopsychological constitutional system underscore the oddity of his structuralist project. In section 12 he writes:

> Whitehead and Russell, through deriving the mathematical disciplines from logic, have shown in all strictness that mathematics . . . only makes such structural claims. The situation, however, seems to be completely different in the empirical sciences: *an empirical science must know whether it speaks of persons or villages*. This is the decisive point: *empirical science must indeed be able to distinguish between such different objects* [Gebilde]; it does this at first mainly through definite descriptions with the help of other objects [Gebilde]; finally, however, the definite description takes place through structural descriptions only.

Here we see again Carnap's epistemological differences with Russell. The point is not that the logicist reduction was an epistemological success because it reduced all mathematical concepts to ones with which we have acquaintance (and, in this way, provided a preeminent example of constitutional epistemology to be taken over into empirical science). Rather, logicism acquires its epistemological point by reducing mathematical concepts to purely formal ones, where logic is simply the locus of form. It is this connection between objective scientific concepts

and pure form that we seek to extend to empirical science and capture in the constitutional system.

Thus, Carnap seems to assimilate the empirical sciences to the formal sciences here. That is, it seems that the goal of constitution reaches in both cases back to the purely structural, as given in logic. It would seem, then, that the structuralist project divorces science entirely from the empirical realm – that, in Carnap's hands, empirical science achieves objectivity by, in the end, severing its ties to empirical reality in just the way mathematics and logic do.

Care must be exercised here, however. For 'reality' is itself a concept that has a determinate, objective sense for Carnap only by finding its place within the constitutional system (§§170–8). Thus, presenting the interpretative problem as one of the connection between the constitutional system and empirical reality threatens to beg rather large questions against Carnap's understanding of what he is doing. It is more useful to think about what role logical structure is meant to play in Carnap's project for the objectivity of empirical judgment. The question then becomes whether he can still maintain a clear distinction between logico-mathematical concepts and empirical ones, given the objectifying role he envisages for logic. For surely Carnap does want to maintain a distinction between logic and empirical science, even if this difference cannot be adequately captured in terms of a philosophically primitive understanding of concepts "connecting to empirical reality."

More appropriate for our task, then, is to consider Carnap's project with respect to objectivity in the *Aufbau* in relation to similar projects in the philosophical air at the time. Here, we can gain a degree of understanding of the epistemological point of Carnap's project by looking at a disagreement that arose among the neo-Kantians, to which Carnap refers in section 12. Members of the Southwest school of neo-Kantianism, especially Heinrich Rickert, argued against the unity of science by maintaining that the type of concept formation used in the natural sciences was incapable of capturing the particularities of individual objects. A representative selection from Rickert's seminal book *The Limits of Concept Formation in the Natural Sciences* (1929/1986, pp. 38f.) asserts:

> We must realize the significance of the fact that all natural scientific or generalizing concept formation ignores the concrete reality of and individuality of unique empirical reality . . . A natural scientific representation that proceeds by generalizing no longer refers to this or that distinctive real thing. On the con-

trary, it abstracts from all properties that constitute objects as these distinctive realities, the very properties that are essential to their status as real entities.

Thus, he claimed that any genuine science of the individual must use a different type of concept formation, one based in empathy, or *Verstehen,* which captured the individual in its particularities and was employed by the human and cultural sciences.

This objection to natural scientific concept formation need not rely on a realist account of how concepts are constituted from experience. That is, it need not be taken to presuppose that fully determinate objects are given as such in experience and that natural science forms concepts via abstraction. Rather, the question is wholly one of whether schematizing, scientific concept formation is capable of constituting individual objects of knowledge. If we lack the ability to capture the particularities of individuals via a notion of concept formation based in logico-mathematical form, we would need to endorse a different notion of concept formation for individual concepts. This is just what Rickert seeks. The challenge to a project such as Carnap's would then be to show how formal logical means of definition suffice to capture determinate, unique empirical objects.[3] What Carnap must provide is a method of concept formation that succeeds in capturing the individuality of the concepts used in the sciences generally.

On this issue, Carnap remarks (§12):

Recently the demand for a "logic of individuality," that is, for a method of conceptual manipulation that is fair to the particularity of individual given things [Gegebenheiten] and does not seek to comprehend these through step by step confinement in species concepts (classes), has been repeatedly raised (in connection with the thought of Dilthey, Windelband, Rickert) . . . I only want to point out that the concept of structure in the theory of relations forms a suitable basis for such a method.

He favorably cites Ernst Cassirer's (1910/1953, chap. 4, §9) response to Rickert here as well. Carnap follows Cassirer in claiming that Rickert has simply misunderstood the nature of the scientific concept as a class rather than relational notion. He returns to this issue in the discussion of the form of the basis in section 75, where he writes: "Cassirer has

3 'Object' is being taken here in the widest sense, in accordance with Carnap's admonition of §1, as I noted in Chapter One. The point is not to distinguish concepts from objects but to indicate how precise, determinate significance accrues to any scientific concept or object. This explains the fluidity of my mode of expression here; given the argumentative context, this is scarcely avoidable.

shown that a science that has the goal of determining the individual through lawful connections without losing its individuality must use, not class ('species') concepts, but relational concepts, since these can lead to the formation of series and, thus, the erection of systems of order."[4]

Cassirer's response to Rickert derives from his general view that the form of the scientific concept is the functional concept of mathematics, not the generic, abstractive concept of Aristotelian logic.[5] Rickert, however, takes the particularity of the individual empirical object to be captured by the qualitative particularities of our experience or empirical intuition. Against this, Cassirer claims that the objective notion of individuality is found not in fleeting and private experience, but in the unique place of the object in a system of scientific laws. He writes (1910/1953, pp. 224f.):

It is not evident that any concrete content must lose its particularity and intuitive character as soon as it is placed with other similar contents in various serial connections, and is in so far "conceptually" shaped. Rather the opposite is the case; the further this shaping proceeds, and the more systems of relations the particular enters into, the more clearly its peculiar character is revealed . . . The individual in its peculiarity is threatened only by the universality of the blurred generic image, while the universality of a definite law of relation confirms this peculiarity and makes it known on all sides.

This kind of functional connection first determines the identity of the object. As a functionally unique member of a system of relations, the object is rationally constituted. Cassirer writes (1910/1953, p. 149):

The chaos of impressions becomes a system of numbers . . . It is true that, in the symbolic designation, the particular property of the sensuous impression is lost; but all that distinguishes it as a *member of a system* is retained. The symbol possesses its adequate correlate in the *connection* according to law that subsists between the individual members, and not in any constitutive part of the perception; yet it is this connection that gradually reveals itself to be the real kernel of the thought of empirical "reality."

As we saw in the passage from section 75 of the *Aufbau* quoted earlier in this section, Carnap thinks that Cassirer has understood the methodological situation quite well. Where Carnap differs from Cassirer is in

4 This remark is again explicitly within a discussion of the controversy between Cassirer and Rickert.
5 We shall examine Cassirer's views at greater length in Chapter Five.

his commitment to the logic of *Principia Mathematica* as the operative logic of relations that provides the technical resources to carry out his own project. Carnap, thus, has a precise and powerful formal logical tool that he employs to formulate Cassirer's insight and sketch of how the process can go forward. The definitional tool he describes that will determine the individual via its place in the relations in which it stands is the *purely structural definite description* (hereafter, *PSDD*) (§12).

Before we look at Carnap's example of a purely structural definite description, we can get a good sense of the conditions it is meant to fulfill by looking at a related project to which Carnap objects – the project of "implicit definition" of scientific concepts, as exploited in Moritz Schlick's (1925) *Allgemeine Erkenntnislehre* (General theory of knowledge).[6] Schlick essentially takes up the challenge Rickert lays down by seeking to extend David Hilbert's technique of implicit definitions to the concepts of empirical science. Carnap finds this approach to be wanting in several respects, and his objections can sharpen our understanding of the point of the project of giving purely structural definite descriptions.

In the *Allgemeine Erkenntnislehre*,[7] Schlick notes the importance of interconnected networks of laws of nature for the understanding of the world and the prediction of future events, that is, the epistemic importance of the world of physics. He also puts forward the thesis that sensory images – for which he frequently (and confusingly) uses the favored Kantian term "intuition" – are too "fleeting and variable" (Schlick 1925, §6) to serve as a basis for scientific knowledge.[8] Thus, in this work Schlick sets himself the task of clarifying and interpreting scientific knowledge in a way that does not view sensory experience as the locus of such knowledge. He, too, emphasizes the structure of science as a whole to explain how science both yields and employs completely precise concepts.

6 For more on Schlick's project in *Allgemeine Erkenntnislehre*, see Oberdan (1990, 1993, 1996), Lewis (1991), and Turner (1996).

7 All quotations from *Allgemeine Erkenntnislehre* are taken from the Blumberg translation.

8 We have already seen Rickert using the term "empirical intuition" in this context, and it is clear that Schlick has only empirical intuition in mind. Strictly speaking, it might be more felicitous, from the point of view of keeping Kantian distinctions straight, for all these philosophers to have used the term "appearance" – the undetermined object of intuition – or "impressions" (*Empfindungen*) here. For Kant, empirical intuitions presuppose the forms of pure intuition and are not fleeting and variable in the appropriate way. Here, as elsewhere, care must be taken not to confuse the issues by presupposing that experience is a mere play of sensations. Precisely this issue marks a distinction between empiricism and Kantianism.

Schlick's connection with the challenge of the Rickert wing of neo-Kantianism is clear from the following account he gives of what he is doing (1925, §3): "There is in fact only one method that can yield scientific knowledge in the strictest, most genuinely valid sense and thus satisfy the *two* conditions under discussion: to determine the individual completely and to achieve this determination by a reduction to that which is most general." The method he endorses is the mathematical method of implicit definition, developed and exploited by David Hilbert (1899) in his work on the foundations of geometry. This method is, according to Schlick, our way of capturing the individuality of the elements of empirical reality through general concepts.

Implicit definition, or *definition by axioms*, proceeds via the laying down of certain axioms in which appear concepts that are not independently defined and stating that those concepts are simply whatever makes those axioms true.[9] Thus, in his formalization of geometry, Hilbert said that the concepts such as 'Point,' 'Line,' and 'Plane' that occurred in his axioms were whatever make those axioms true. In this way, the concepts used in the axioms are implicitly defined by general truths that hold of these concepts. For example, the concepts 'point' and 'line' are partially defined by the general truth that "For any two distinct points there is a line such that those points lie on that line."

Schlick sees this mathematical method as providing a response to the demand that Rickert makes. Implicit definition determines concepts on the basis of general truths about those concepts, and it does so completely. The second conjunct is true because the words for the concepts are stipulated as designating whatever makes the axioms true. There is no further constraint placed on the concepts. Hence, there is no appeal to intuition here, whether of an empirical or *a priori* nature; our concept 'line' is not given through lines constituted by us or given to us in intuition.

For Schlick, rigorous natural science must also define its concepts via implicit definition if it is to "determine concepts completely and thus to attain strict precision in thinking" (1925, §7). But on the face of it, the

9 It is, of course, more usual now to express this point in what Carnap termed "the formal mode of speech": The terms occurring in the axioms refer to whatever makes the axioms true. This became the standard only after the metamathematical and metalogical turns of the late 1920s and early 1930s, however, and is not how either Schlick in his book or Hilbert in his early work on geometry presented the idea.

concepts of natural science cannot simply be defined by the fact that they fulfill this or that consistent axiom system. One and the same axiom system can, for example, give the structural characteristics of a sequence of railroad stations and a sequence of telephone connections. But these things are different from the point of view of natural science. That is, implicit definition determines concepts in relation to one another but does not determine the connection between such concepts and empirical reality at all. Schlick himself puts the matter this way (1925, §7):

> Implicit definitions have no association or connection with reality at all; specifically and in principle they reject such association . . . A system of truths created with the aid of implicit definitions does not at any point rest on the ground of reality. On the contrary, it floats freely, so to speak, and like the solar system bears within itself the guarantee of its own stability.

This is a large unanswered problem in Schlick's thought on scientific knowledge in *Allgemeine Erkenntnislehre.* Ultimately, he seeks to mitigate it by claiming that what is at stake here is the precision of scientific thinking and that this is where the structural aspects of implicit definition are exploited; the precision of scientific theorizing is the same as the precision of mathematical thinking: Both are purely formal. Thus, if, in the end, Schlick must allow for empirical intuition to creep back into science – in order to provide some indication as to which of many possible axiom systems is true of empirical objects – this yields no worse a consequence than the well-known empiricist position that the truth of judgments in the empirical science "is not absolutely guaranteed" (1925, §11). The precision of science is saved, but we can never know with absolute certainty whether the axiom systems of theoretical science capture the structure of the world.

This answer, however, simply changes the question from what Schlick claimed it was in the passage just quoted. This answer requires that objects be given in empirical intuition as such and thus sets aside the project of defining objects via their conceptual interconnections. In Schlick's original formulation of the problem, and for Carnap, the relation of structure and science goes deeper; it is to explain not the precision of science but its objectivity – its ability to make claims about the world at all. On Carnap's view, if we eventually had to rely on the individual's subjective experience in defining scientific concepts, those concepts would themselves be irreducibly subjective. Science would altogether cease to be about an objective world that is identical for all of

us. That is to say, Schlick's eventual endorsement of implicit definition, which defines concepts through the assertion of various judgments, leaves open the possibility that those judgments are not precisely true of anything in reality. For Carnap, until and unless we have precisely and structurally defined concepts, the making of judgments in any significant sense is impossible. The issue of the truth or falsity of any particular judgments, including those of our favored axiom systems, is secured only by our ability to make objective judgments at all.

Carnap's logicism provides him with both the means to criticize a project of implicit definition such as Schlick's and an alternative paradigm of concept formation. Carnap argues for a logicist rejection of Hilbert's formalism and, hence, also of the method of implicit definition, even within the mathematical sciences. The logicist program in mathematics differs from the formalist program in that, although both provide axiom systems from which (it was thought) all mathematical theorems could be derived, the logicist alone provided definitions of the mathematical concepts that were symbolized in the axiom system. Thus, the Russellian account of arithmetic differs from that given by the Peano axioms in that *Principia Mathematica* also provides definitions of the basic concepts of the Peano axioms, for example, 'number' and 'successor.'[10] It is this additional feature which allowed Carnap to conclude that the logicist program alone provided an explanation of the *objectivity* of arithmetical knowledge.

Carnap presented his reasons for this assessment of logicism and formalism in his 1927 essay entitled "Eigentliche und uneigentliche Begriffe" (Proper and improper concepts; hereafter *EUB*). The principal focus of this essay was a discussion of proper (*eigentlich*) concepts and their relation to improper (*uneigentlich*) concepts. "Proper concepts" are those which are explicitly defined within a constitutional system, whereas "improper concepts" are those only implicitly defined via the axioms of this or that axiom system. Within the domain of proper concepts, Carnap further distinguished empirical concepts from formal concepts. "Formal concepts" are the concepts of logic and mathematics (as given in *Principia Mathematica*) whereas "empirical concepts" are (EUB, p. 356) "the concepts of real objects."

Carnap was at pains in his essay to establish that the concepts of natural science are all proper concepts and yet to explain the nature and

10 There are many ways to formulate the Peano axioms. They can be formulated such that 'number' and 'successor' are the only primitive concepts, however. See Russell (1919, chap. 1).

usefulness of axiom systems as implicit definitions of improper concepts. He made three distinctions between proper and improper concepts. First, proper concepts do, whereas improper concepts do not, fulfill the law of *tertium non datur*. Within the framework of type theory, the claim that proper concepts fulfill the law of *tertium non datur* amounts to the following: Consider a concept represented by the symbol ϕ. Then for any object, α, such that $\phi\alpha$ is meaningful, $\phi\alpha$ is either true or false. That is, any object of appropriate type either falls under the concept or it does not. As the concept represented by ϕ is also of some particular type, it can serve as an argument for predicates of higher type. So, since *tertium non datur* also holds for these concepts, we arrive at the following fact about proper concepts: For any concept, Φ, such that $\Phi\phi$ is meaningful, $\Phi\phi$ is either true or false.

This last claim is, of course, untrue in general of concepts implicitly defined via axiom systems. In order to implicitly define a concept, an axiom system need only be consistent. This very weak condition on the axiom system makes the last claim fail for a large class of improper concepts. Carnap's example (EUB, p. 363) is as follows:

Axiom System I:

1. The field of R has three members.
2. R and the ancestral of R are irreflexive.
3. R is intransitive.

Axiom System I "defines" the concept 'R.' The models of Axiom System I and, hence, of R allow, however, of two nonisomorphic structures. The arrow diagram for R can be of either of the following types:

$$\circ \rightarrow \circ \rightarrow \circ \qquad \circ \leftarrow \circ \rightarrow \circ$$

But the structural properties of R are different (obviously) in these two diagrams. In particular, since Axiom System I allows these two models, the sentence "R is one-to-one" is meaningful but neither true nor false. R, as defined by Axiom System I, neither has nor lacks the property of being one-to-one.

What is crucial to the argument here is that one-to-oneness is a structural property of relations. As such, it must be meaningful to ask of a given relation if it is one-to-one or not, and, thus, every proper relation either has this property or not. But relations specified by some consistent axiom systems are not specified sufficiently to determine whether

they have this structural property. Thus, these axiom systems surely do not count as definitions of the relations mentioned in them.

The second distinction between proper and improper concepts that Carnap finds is the following. He claims that (EUB, p. 367) "It belongs to the essence of a proper concept that for every object it is in principle decidable whether that object falls under that concept or not; in fact with sufficient knowledge of the object this decision is also practically feasible." This, however, is not the case with concepts improperly defined by axiom systems. Consider Carnap's example of the natural numbers, as defined by the Peano axioms. This axiom system allows of many interpretations, including any physically existing infinite sequence of spheres (if there is any such series). The question, Is this sphere a number? is, however, meaningless in abstraction from the entire interpretation. If this sphere is part of the infinite sequence of spheres which forms an interpretation of the axiom system, then, within this interpretation, the sphere is a number; if we interpret the axiom system using an interpretation based on spatial or temporal points, then the sphere is not a number. No amount of knowledge that we can gain of the sphere will, however, provide a univocal answer to the question as to whether the sphere is a (Peano) number. The question therefore lacks an answer, and *tertium non datur* fails here as well.

This last difference between proper and improper concepts is related to (Carnap [EUB, p. 370] calls it a "symptom" of) the final and ultimate distinction Carnap claims for these two kinds of concepts. Carnap states (EUB, pp. 371–2):

> The sign of an improper concept is the sign of a variable which is based on a certain axiom system in such a way that the sentential [satzartigen] signs in which it appears are to be made into proper sentences through a specific kind of supplementation to the axioms of the axiom system.

Carnap puts forward the view that improper concepts are mere placeholders – variables – through the use of which one can become familiar with the formal consequences of certain axioms. Thus, the theorems of Peano arithmetic, for example, are best thought of as abbreviated forms of long, purely logical theorems in conditional form beginning with a universal quantifier binding each nonlogical symbol and having as antecedent the conjunction of the axioms and having as consequent the putative theorem.

This way of viewing axiom systems and the implicitly defined concepts they contain gives substance to the brief remarks Carnap makes about implicit definition in section 15 of the *Aufbau*. Carnap there states that implicit definition can lead only to analytic statements about the (improper) concepts so defined. This is certainly not sufficient for a system of scientific concepts. Carnap points to the need to define the concepts of science such that contentful, empirical statements containing the signs for those concepts can be made. In "Eigentliche und uneigentliche Begriffe," Carnap puts the point this way (p. 372):

> Empirical concepts are constituted step-by-step in the systematic constitution of [our] knowledge of reality. As a member of this constitution every empirical concept has immediate relation to reality. In contrast to this, improper concepts chiefly float, so to speak, in the air. They are introduced through axiom systems, which are themselves not in immediate relation to reality.[11]

So what Carnap needs in his constitutional system for empirical concepts is a method of definition which is formally sufficient for a step-by-step constitution of the entire system of scientific concepts, but one which is superior to implicit definition in that it maintains connection between the defined concepts and empirical reality – or, more precisely, that allows for synthetic, empirical knowledge. This is, of course, not necessary for the logicist constitutional system for mathematics given in *Principia Mathematica*. Whitehead and Russell's system shows the concepts of mathematics to be proper concepts, but they are also shown to be formal rather than empirical concepts. Thus Carnap needs to go beyond the system of *Principia Mathematica* in the definition of empirical concepts as well.

It is the need to preserve empirical knowledge and synthetic *a posteriori* judgments that constitutes the requisite connection with empirical reality. Thus, Carnap's use of metaphysical language in the passage just quoted is wholly metaphorical. It should be clear that what is doing all the philosophical work for Carnap throughout his discussion in this essay is simply the status of *tertium non datur* as a logical truth. Its apparent failure for implicitly defined concepts indicates that concepts are not adequately specified by such axiom systems. What is, therefore, necessary for the well-foundedness of both ϕ and α is that we can make sense of $\phi\alpha$, and thus not-$\phi\alpha$ *because* "$\phi\alpha$ or not-$\phi\alpha$" is a logical truth and,

11 This passage appears to contrast improper concepts with empirical concepts. Carnap's point is, however, that implicit definition of empirical concepts leaves them without relation to the empirical realm. Formal concepts, even when properly defined as in *Principia Mathematica*, still lack this relation to empirical reality, but that is not a problem, as we shall see later in the chapter.

ipso facto, meaningful. The contact with "reality" requisite for empirical concepts is just this requirement of sense, which is driven by adherence to the logical truth of *tertium non datur*. In the case of empirical concepts, this means finding a sense for empirical judgments involving those concepts. Carnap hopes to fulfill all these desiderata in his procedure of purely structural definite descriptions. It is to this procedure that we now turn.[12]

PURELY STRUCTURAL DEFINITE DESCRIPTION

A "purely structural definite description" (PSDD) picks out an object uniquely on the basis of structural features of the relations in which it stands to other objects in a domain. It relies only on the structural features of the relations, that is, on those features of it that are preserved in the arrow diagram or list of n-tuples (where the objects related by the relation are given arbitrary designations). For relations among empirical objects, these structural features are typically only empirically known. Such a definite description, according to Carnap (§13), has the advantage that we do not have to rely ultimately on experiential ostension of any object in the domain as the ground of the sequence of definite descriptions for the objects in that domain. Thus, the purely structural definite descriptions fulfill the desiderata we have for the definitions of empirical objects for objective science: They rely only on the structure of experience; it is an empirical matter that experience has a structure that permits them; and no prior determination of any of the objects in immediate acquaintance is presupposed by or expressed in the definitions.

We can illustrate the essential features of purely structural definite descriptions through a simplified version of the railway example Carnap provides in section 14. Imagine a city with a very simple subway system that has the following structure: There are two lines, one running east and west and the other running north and south. The two lines intersect at exactly one station, where passengers can make free transfers from one line to the other. Let us suppose that this station is called "City Center." Suppose further that the eastern terminal of the

12 Thus there is an inaccuracy in Friedman's (1987, p. 542, n21) claim that Carnap "endorses" Hilbertian implicit definition in §15. Although Carnap does positively claim "scientific importance" (as shown by Schlick) for such definitions, the bulk of this part of §15 is meant to underscore the advantages of Carnap's definitional method over implicit definition. Compare also Carnap's remarks at §121 and EUB, p. 373.

system is six stations from City Center, the western terminal is five stations from City Center, the northern terminal is four stations from City Center, and the southern terminal is two stations from City Center. Call these stations, respectively, "East," "West," "North," and "South." The relation of nextness between stations is the important one for the construction. Let us call it "R." This relation has enough structure that we can uniquely define all the stations from it via Carnapian PSDDs. We can, through these definitions, then introduce names for the stations as defined terminology into our language.

The feature of the structure of R that allows the formation of PSDDs in this case is the existence of stations with differing numbers of neighbors. The vast majority of the stations have exactly two neighbors. The four terminal stations, however, have only one neighbor each, and one station (City Center) has four. This last station gives us our easiest entering place for the PSDDs. Using the abbreviation in (1), we can use the truth of (2) to introduce the term "City Center" into our language via (3):

(1) $\Phi x =_{df} (\exists y)(\exists z)(\exists w)(\exists v)[(Rxy \& Rxz \& Rxw \& Rxv \& y \neq z \& y \neq w \& y \neq v \& z \neq w \& z \neq v \& w \neq v) \& (\forall u)(Rxu \rightarrow (u = y \vee u = z \vee u = w \vee u = v))]$

(2) $(\exists x)(\Phi x \& (\forall y')(\Phi y' \rightarrow y' = x))$

(3) $\text{City Center} =_{df} (\iota x)\Phi x$

With (3) in hand we can use structurally specifiable differences to distinguish the terminal stations. The crucial fact is that the chains of the R relation are of different length for each terminal station. For example, South is only two stations from City Center. This fact can be captured within the bounds of purely structural definite descriptions in the obvious but long-winded ways: There is one and only one station with exactly one R-neighbor and which stands in relation R to some station that stands in relation R to City Center. The remaining stations can then be defined by the lengths of R-chains leading to them from the defined stations. Once we have all the terminals defined we can define all the other stations via the length of the R-chains from them to City Center and the terminals. All difficulties are thereby resolved.[13]

Definition (3) meets the requirements of a purely structural definite description as Carnap presents the notion in section 13. It uniquely picks out a single object from the field of the relation R that fulfills a condition, Φ, which in turn is specifiable in terms of R and logical terms

13 The details are left as an exercise in tedium for the insomniac reader.

alone. Since R is not itself an object in the domain in question, but rather a relation over that domain, there is no problem induced by the fact that it appears in the definition. Similarly, no object in the domain is ostensively pointed out or introduced as part of the primitive terminology of our language. Finally, it is, of course, only the (in our case, imagined) empirical fact that this system has a neighbor relation of this structure that allows these definitions to work.[14]

Of course, if the relation(s) chosen do not have the structural characteristics necessary for structurally specifying the individual objects in their domains, this is not simply the end of the matter. In such cases, Carnap counsels us to expand the number of relations we look at in trying to come up with the structural definite description: Start with further geographical relations, and expand the number and kind of relations until each station is uniquely determined. Any subjectively different stations, to continue the example, which are indistinguishable even after we have considered all relations from all branches of science among them must be deemed objectively indistinguishable "not only for geography, but for science in general" (§14). Thus there is no *a priori* guarantee that this method will meet with success, but it alone can guarantee objectivity. In section 13, Carnap goes so far as to allow that even "the total domain of all objects of knowledge" may not in fact be ordered in a system of purely structural definite descriptions (constitutional system) but says "it is a necessary presupposition of the possibility of an intersubjective, purely rational science" that such a system is possible.

The type theoretic structure of logic and the existence of relations among relations provide the solution to a related difficulty. Carnap is faced with the problem of specifying all the concepts of science purely structurally. But it is clear that relations considered in isolation can share structure. Thus, a simple structuralist account would not be able to distinguish, for example, any two equivalence relations defined over domains of the same cardinality when these relations are considered in isolation. Carnap's guiding idea is that by embedding these relations and the objects over which they hold in a rich enough structure of further relations, each of the relations will be purely structurally defin-

14 A system with only two stations, each of which is the other's only neighbor, is obviously too structurally impoverished to yield any system of PSDDs for the stations. We would have to bring different relations and, presumably, a larger object domain of which the stations were only a small subclass to bear before we could get the definitions going.

able. To use Michael Friedman's terminology (1987, p. 528), such "locally" structurally identical relations can be distinguished by their places in the overall "global" network of relations. Thus, Carnap exploits the idea that relations over objects at some type level are themselves individuals in the domain of other relations of higher type to achieve the rich structure of relations needed to achieve a constitutional system for total science.

It is this strategy of uniquely determining all the objects (concepts and relations) of science that grounds the project for achieving objectivity via PSDDs. Each relation is ultimately uniquely determined by its position in the whole nexus of relations. This is meant to answer an objection one might bring to our example of a PSDD in definition (3). We said that this definition meets the requirements for a PSDD because no element of the domain was ineliminably mentioned in it. But, of course, the relation, R, itself is ineliminably mentioned in the definition. Thus, definition (3) counts as a PSDD within a constitutional system for total science only on the presupposition that R itself has already been or subsequently will be given a PSDD of its own. Nothing Carnap has said in his example of section 14 and nothing that I have said in following this example have lent any plausibility to the claim that we can do this.

Again, we must be careful in what we are asking Carnap for here. Surely Carnap is not in a position to provide a blanket *a priori* guarantee that this can be done in any particular case. The method of giving PSDDs for relations via their unique place in the whole network of relations must, however, be possible if objective science is possible. Thus, it is a methodological presupposition of the constitutional project that the method will work for the epistemologically ordered constitutional system for the totality of science (or indeed any such total constitutional system). How it can proceed is no more difficult to understand than it is to understand how the PSDDs for the object domain of R were formulated. Thus, although the example does not yet show how elimination of essential reference to particular relations is possible – doing that would require that the entire constitutional system already be in place – it does show how the constitutional system can proceed given that it has gotten started. This is to be achieved by not merely quantifying over relations but by picking them out by definite description according to their purely structural but empirically given relations within the whole network of relations. That is, the uniqueness of the described relation is logically presupposed in the definition but is only empirically ascertainable. The empirical nature of the guarantee of the uniqueness of reference of the structural definite description is meant to

give content to the idea that the constitutional system is a constitutional system for empirical concepts, as opposed to the formal concepts of mathematics. This finds its expression in Carnap's distinction between logical and empirical theorems within the definitions of the system.

There is, of course, more to be said here. There are serious worries still about the viability of the structuralist project. These worries are manifest in the *Aufbau* itself in sections 153–5, in which Carnap attempts to define away the basic relation itself. Here the tension between objectivity as form and the empirical basis of knowledge comes out with clarity. The discussion of these sections must wait for its proper place in the outline of the constitutional system, however.

QUASI ANALYSIS

There is one other general technical problem that Carnap's conception of the constitutional project entails, regardless of the constitutional order chosen. Carnap conceives of the basis of any constitutional system as consisting of one or more relations holding over a domain of primitive elements. The basis elements have no primitive qualities; that is, the relations are not supplemented by primitive property descriptions. Thus, methodologically, the basis elements are free of qualities. Indeed, the qualities that hold of the basis elements are among the things that must be constitutionally defined. So, he needs to devise a general method of constituting properties from relation descriptions. His solution is the procedure he terms "quasi analysis."[15]

The point of quasi analysis is to allow the construction of properties of individual basis elements from the relations that hold among those elements. Formally, therefore, the procedure of quasi analysis is an extension of the method of abstraction used by Whitehead and Russell in their work on the foundations of mathematics. Perhaps the most famous use of their method of *abstraction* (or *analysis*) is as part of the logicist definition of the natural numbers. Consider sentence (*):

(*) The earth has (exactly) one moon.

Logicists worried what meaning could legitimately be assigned to such a sentence. Very roughly, the idea was to think of (*) as expressing the

15 The process of quasi analysis has engendered a good deal of the secondary literature on the *Aufbau*. I have used Goodman (1953) throughout the following discussion. For other literature, see, e.g., Kleinknecht (1980), Lewis (1969), Mormann (1994), and Proust (1989 [1986], §4, chap. 2).

claim that the predicate "is a moon of the earth" is satisfied by exactly one object, that is, that the class of objects satisfying the predicate is a one-membered class. This, in turn, helps the logicist account of number claims only insofar as the notion of 'one-membered class' can be explicated without a primitive notion of 'one.'

This is where the method of abstraction comes in. The idea is that a primitive relation of equinumerosity can be brought in to aid the definition of the number of a class. Consider a relation of equinumerosity for classes which holds just in case there is a function that maps each element of one class onto exactly one member of the other. The notion of a one-to-one function can itself be understood in terms of the existence of a relation and the existence and uniqueness of relata of that relation. Thus, it can be expressed in wholly logical terms. This will do the trick for the logicists. Equinumerosity is a logical notion that either obtains or fails to obtain between any two classes.

The method of abstraction now allows particular equivalence classes of the relation of equinumerosity to be formed. For example, we can define the class "zero" as the class of all classes equinumerous with the empty set. (The empty set is itself definable as, say, the class of all things not identical with themselves, that is, the extension of the predicate "not identical with itself.") "One" can then be defined as the class of all classes equinumerous with zero (since zero turns out to be simply the class containing only the empty set). The meaning of the asterisked sentence is then given by the following sentence:

(**) The extension of "is a moon of the earth" is an element of one.

The crucial notion for Carnap, if not for the particular purposes of the logicists in this case, is that here we have a property of an object (in this case, a class) defined from a relation over such objects (in this case, the relation of equinumerosity between classes).

The extension of this to the empirical realm is clear enough. To take the particular example of Carnap's "elementary experiences" (*elex*), we want, for example, to be able to claim

(#) Elementary experience, $elex_{23}$, has a red dot in a particular point of the visual field.

The idea is to try to construct from the basic relation among the elementary experiences the class of the elexes that have a red spot at that

point of the visual field. The meaning of this sentence is then that $elex_{23}$ is an element of the class of such elexes. Carnap has no other way to make sense of any property ascription to the elexes, since they are understood to be primitive and, thus, without properties somehow independent of the constitutional project itself. Carnap calls such constructed properties "quasi properties" and the process by which they are constituted "quasi analysis."

Formally, then, quasi analysis is simply an extension of the notion of analysis. The distinction Carnap draws between analysis and quasi analysis is epistemological. He introduces the notion of analysis with these words (§70): "In proper analysis, we are concerned not with propertyless points or indivisible unities, but rather with objects that have various constituents (or characteristics). Analysis consists in inferring these constituents, which are at first unknown, from other data, for example, from a relation description." Quasi analysis, on the other hand, is not a process by which one infers properties of complex objects from relations among them. It is, rather, a process of constituting quasi constituents or quasi qualities from relation descriptions holding over propertyless points or indivisible unities. Thus, quasi analysis is, Carnap tells us (§74), "synthesis in the linguistic garb of an analysis." The properties that the basis elements are eventually said to have are not analyzed from complex basis elements but are logical constructions from these elements, classes of the elements derived from the basic relations given at the start of the system. Thus, analysis and quasi analysis are formally identical procedures for Carnap. They differ only in their constitutional and epistemological roles. Analysis presumes an antecedent fact of the matter about the properties of complex objects and tries to recover such properties from relations among those objects. Using formally identical techniques, quasi analysis seeks to constitute classes that play the role of the properties for objects that are constitutionally primitive. That is, in quasi-analytic contexts there is no independent or antecedent sense in which the objects standing in the relations have properties; they have only those properties that are definable in a quasi-analytic way.

The formal identity between analysis and quasi analysis aids with the exposition of Carnap's formal concerns. In situations such as the one just discussed, analysis consists in the definition of equivalence classes from a given "equivalence relation." Any relation that is reflexive, symmetric, and transitive is an equivalence relation; equinumerosity, for example, is an equivalence relation. Carnap's quasi analysis cannot rely on the basic relation(s) being equivalence relation(s), however. He needs a more general procedure, since his relations might be rather differently structured. There are two ways in which Carnap thinks that the method of analysis must be extended if it is to be applicable to the basic relation of the system actually presented in the *Aufbau*. These extensions are necessary in order to exploit the structure that a genuine recollection of similarity relation is likely to have.

Let us recall the sort of thing that Carnap must try to construct. He needs to construct, for example, the colors in the visual field of a given elementary experience. If it were the case that the visual field contained only a single color for any given elex, then he could perhaps (somewhere not too far up the constitutional order) simply use the method of analysis to create the equivalence classes of a color identity relation for the visual field. He cannot, however, do that if (as seems reasonable) the visual field can contain more than one color. If this is the case, then the only way to define a class of elexes that have, say, a particular shade of yellow somewhere in the visual field is not via any equivalence relation but by what Carnap will call a *part identity relation*. This relation captures the idea that a given object is partially (qualitatively) identical to another. For example, the class of elexes with a specific shade of yellow in the visual field might contain one elex that also has a shade of blue in the visual field and a second one that contains a shade of red in the visual field. These two elexes are part identical to one another – they both contain that yellow shade – but the first will also be part identical to other elexes to which the second is not. In other words, it is clearly the case that quasi analysis must be applicable to nonequivalence relations. A part identity relation is a similarity relation only; it fails to be transitive.

We can understand Carnap's attempted solution to the problem of extending the construction of classes from equivalence relations to similarity relations through a consideration of examples drawn from Nelson Goodman's *Structure of Appearance* (1953). Here, I shall follow Goodman in giving examples that Carnap would think of as following the method of analysis rather than quasi analysis. I shall not lose sight of the distinction between analysis and quasi analysis, however, and I shall return to it when responding to Goodman's philosophical concerns with quasi analysis.

Let us consider a world of six individuals and three properties. Each of these properties is a color; they are, respectively, a particular shade of red, blue, and green and will be designated *r*, *b*, and *g*. We shall designate the individuals by the first six arabic numerals for the natural

numbers (1 through 6). To say that object 4 is green and only green, I shall write "4. g."

Before we get to more complicated situations, I shall start with an easy example in which each object has one and only one color. Such a world might look like this:

1. r	2. b	3. g
4. b	5. b	6. r

Proper analysis is a process whereby we would recover these properties of these objects from a relation holding among them. For example, we might have the relation of color kinship – where one object is color akin to another just in case they have a color in common.

Given the extensionalist framework within which we are working, to recover the properties amounts to no more than giving a procedure that recovers the appropriate classes of the individuals. In our simple case the classes we want to recover are the three classes {1,6}; {2,4,5}; and {3}. Similarly, in this case, the relation of color kinship has a very simple structure. It is reflexive, symmetric, and transitive over the field; hence, it is an equivalence relation. Its ordered pair list is:

⟨1,1⟩; ⟨1,6⟩
⟨2,2⟩; ⟨2,4⟩; ⟨2,5⟩
⟨3,3⟩
⟨4,2⟩; ⟨4,4⟩; ⟨4,5⟩
⟨5,2⟩; ⟨5,4⟩; ⟨5,5⟩
⟨6,1⟩; ⟨6,6⟩

The way to recover the appropriate classes is simple: A color class is a class of objects all of which stand in the relation of color kinship to one another and such that there is nothing outside the class that stands in the color kinship relation to all members of the class. The first conjunct disallows classes such as {1,2}: The members of this class do not stand in the color kinship relation. The second conjunct disallows classes such as {2,4}: There is an individual outside this class that stands in the color kinship relation to all the members of the class, namely, 5. The classes we want are the largest classes all of whose members are color akin.

This is a very easy case to understand, precisely because we have an equivalence relation in this case. The color kinship relation divides the field into equivalence classes. These equivalence classes are precisely the classes we are looking for. Thus, in such a case, not only is there no element outside the color class that stands in the relation to *all* the elements inside, but there is no element that stands in the relation to *any* of the elements. This is, however, a special case. Since each individual has one property, the classes constructed from the kinship relation to serve as quality classes do not overlap.

Let us complicate our picture by making our imagined world slightly more realistic. In this example, our individuals can have one or more colors. In such a case, color kinship will now be a part identity relation and, thus, a similarity relation. An example from Goodman has the world consisting of the following individuals:

1. br	2. b	3. bg
4. g	5. r	6. bgr

Given this tally of the individuals, we can list the classes our analytic procedure must deliver: {1,2,3,6}, {1,5,6}, and {3,4,6}. We can also construct the pair list of the color kinship relation:

⟨1,1⟩; ⟨1,2⟩; ⟨1,3⟩; ⟨1,5⟩; ⟨1,6⟩
⟨2,1⟩; ⟨2,2⟩; ⟨2,3⟩; ⟨2,6⟩
⟨3,1⟩; ⟨3,2⟩; ⟨3,3⟩; ⟨3,4⟩; ⟨3,6⟩
⟨4,3⟩; ⟨4,4⟩; ⟨4,6⟩
⟨5,1⟩; ⟨5,5⟩; ⟨5,6⟩
⟨6,1⟩; ⟨6,2⟩; ⟨6,3⟩; ⟨6,4⟩; ⟨6,5⟩; ⟨6,6⟩

Can we reconstruct the classes from the pair list? It seems that we can without trouble through the selfsame idea. The color classes, which Carnap calls *similarity circles* (§70) (owing to complications noted later in this chapter), are again those classes of the elements such that any two elements are related by the relation and no element outside the class is related to everything inside. Indeed, in the given case, this recipe yields exactly the three classes we want.[16] In these cases where transitivity fails, Carnap's idea is that the similarity circles constructed in this way will distinguish between overlaps that do not indicate a shared property and those that do indicate a shared property.

Before considering Goodman's concerns about this aspect of Carnap's project of quasi analysis, let us conclude our exegesis of the formal aspect of the project. So far, the relations we have considered are part identities – relations that indicate that two objects agree in one or more quality. The relation that Carnap has available for quasi analysis in the constitutional system actually outlined, however, is not a part identity; it is a *part similarity*. Two elementary experiences stand in this relation, so to speak, not merely if they agree in a constituent, but also if they share similar constituents.[17] Thus, similarity circles defined by this relation in themselves yield only classes of part similar individuals, and further work must be done to carve the similarity circles into quality classes.[18] Again let us employ Goodman's (1953, pp. 165–8) detailed exposition as our guide.

If we continue to think in terms of analysis, rather than quasi analysis, to motivate the process, we can put the basic ideas as follows: Let us think of a relation of color similarity that holds between two individuals just in case the first has a color that is similar to a color exhibited by the second. (Because this is a symmetric relation, it matters not at all which is considered first or second.) Consider a similarity circle based on this relation: It contains not merely individuals that share a color but individuals that have similar but not identical colors. How do we get the right classes: The ones containing only objects sharing the *same* color? The idea is to look at how the similarity circles overlap. Carnap (§§72, 80–1) considers an analogy with what he terms the "color solid": The color solid is a three-dimensional array of colors, ordered by hue, saturation, and brightness. Imagine this three-dimensional solid sphere (or ball) constituted by small overlapping spheres. For example, imagine the color sphere covered by similarity spheres, each of which captures similar shades of color.[19] How would we define the individual points (colors)? Geometrically, the points are the largest portions of the whole sphere undivided by the small overlapping spheres, and that is how to define them.

This is, then, the guiding analogy: The quality classes are the largest

16 If this is not obvious, Goodman (1953, pp. 158–60) goes through the case in detail.

17 This way of putting the matter gets things backward. The systematic difference between part identity relations and part similarity relations, for Carnap, is given in the structure of the class of similarity circles constructible from these types of relation (as we will see later in the chapter). The terminology is confusing since both part identity relations and part similarity relations are similarity relations, not equivalence relations.

18 The fact that part similarities do not yield quality classes via the process of constructing similarity circles is the reason why similarity circles are so named, and not called "quality classes."

19 In this case the similarity holds over the colors themselves rather than over the colored individuals of the original case.

classes of individuals wholly inside a similarity circle and undivided by the overlapping similarity circles. Think, for example, of the similarity circle "centered" on a particular shade of yellow. The quality class for that shade of yellow would be the largest subclass of that similarity circle undivided by other similarity circles – the circles "centered" on nearby shades of yellow. In practice this amounts to the following first attempt (cf. Goodman 1953, p. 175): A class of individuals, q, is a quality class just in case (i) each element of q is contained in every similarity circle to which any element of q belongs (this bars individuals that find themselves in other similarity circles) and (ii) for every individual not in q there is some similarity circle which contains all of q but not that individual (this gives us maximal classes).

This actually does not work, because of a crucial disanalogy in the two cases (the color solid covered by densely packed color similarity spheres, and the colored individuals arranged in similarity circles based on a color similarity relation). In the case of the color solid itself, a small sphere centered on a particular shade of yellow – say, the yellow in the Swedish flag – does not overlap any sphere centered on any shade of blue. The overlaps occur only with spheres of nearby colors. With the colored individuals, however, there can be overlaps between similarity circles "centered" on widely disparate colors. For example, if among our individuals is a Swedish flag, this will fall into yellow and blue similarity circles. Thus, our condition (i) is too strong; we do not want to throw the Swedish flag out of the class of things exhibiting the shade of yellow in the flag simply because it is in another similarity circle for blue things that most of those yellow things are not in. What we want to do is separate the nearby colors that are lumped together by the similarity relation. We do not care about overlaps from similarity circles from far afield. Carnap distinguishes these cases as "essential overlap" and "accidental overlap" (§§80–1), respectively.

But how are we to distinguish accidental from essential overlap? Carnap (§§81, 112), in essence, proposes to treat accidental overlap as "uncommon." Thus, the first condition of the definition is revised to read (cf. Goodman 1953, p. 176): (i) Each element of q is contained in every similarity circle within which at least half of the elements of q belong. Thus, our Swedish flag is no longer a problem, since the blue similarity circle within which it finds itself contains at most a few more elements of the class of things exhibiting the yellow of the flag.

There are a couple of points about quasi analysis on the basis of part similarity relations that should be noted. First, given that relations do not come tagged as being part identities or part similarities, how do we

decide whether the simpler or the more complicated quasi-analytic procedure should be followed? Carnap (§72) argues that the difference between part similarity and part identity relations is itself a matter of the structure of the initially constructed similarity circles. If, after the construction of the similarity circles in accordance with the simple procedure, there are large multiple overlappings, then we must proceed to the second step. Otherwise, we can treat the similarity circles as quality classes immediately. Thus, the difference between part similarity and part identity relations is not a difference between whether the relations capture qualitative identities or similarities among the individuals; that would contradict the epistemological starting point and constitutional reason for the quasi-analytic process itself. Rather, it is a matter of the structure of the relations alone – a matter exhibited by the structure of the overlapping of the similarity circles constructed from them.

Finally, as Goodman (1953) stresses, the complicated process of quasi analysis on the basis of part similarity does more than simply solve a problem posed by a complicated system of mutually overlapping similarity circles. It also crucially provides an ordering of the qualities, based on the size of the essential overlaps. This ordering of qualities is exploited in the further constructions. For example, classes of qualities have to be linked together and distinguished from other classes of qualities, as when we distinguish tactile from visual qualities. Thus, the more complicated procedure is in accord with Carnap's methodological need for relational structures; the complicated quasi-analytic procedure yields not a mere list but a structure of qualities. Thus, we are not stymied in the search for further constructions.

GOODMAN'S OBJECTIONS TO QUASI ANALYSIS

Perhaps the second most famous objections (after Quine's) to Carnap's project in the *Aufbau* are Nelson Goodman's technical objections to quasi analysis. Thus, I would be remiss not to discuss those objections here. In the end, I shall argue that Goodman's objections are indeed objections to analysis on the basis of similarity relations, but not to quasi analysis on the basis of similarity relations. Goodman does not adequately distinguish between the technical congruence and the epistemological or constitutional noncongruence of these notions in Carnap's project.[20]

20 My response to Goodman is similar to and has been informed by those found in Proust (1989 [1986]) and Mormann (1994).

Recall Carnap's condition for the analytic and quasi-analytic construction of similarity circles on the basis of similarity relations: The similarity circles are those classes each of which is such that any two elements within it are related by the similarity relation and no element outside it is related by that similarity relation to everything inside. Goodman notes that as an analytic process this recipe can easily fail. An example based on one given in the preceding section of this chapter is seen in the following world:

1. br	2. b	3. bg
4. g	5. br	6. bgr

Here, the only difference from the original case is that individual 5 is both blue and red and not merely red. This does not disturb our target class for the quality red, which remains {1,5,6}. But, if we form the pair list for color kinship and then find the similarity circles, this class is not among them. That is because there are individuals not in this class which are color akin to every member of the class, for example, 2. The problem is that all red individuals are also blue individuals. Carnap (§70) calls red a "companion" of blue in such a case. Hence, Goodman (1953, p. 161) calls this the "companionship difficulty."

Goodman also notes a second way in which analysis via similarity circles can fail. Consider the following world, differing from the original only in the colors exhibited by 2 and 5:

1. br	2. br	3. bg
4. g	5. gr	6. bgr

In this case the two classes we should arrive at for blue and red are, respectively, {1,2,3,6} and {1,2,5,6}. Neither of these classes are similarity circles, however. Both of these are proper subclasses of the class {1,2,3,5,6}, which is a similarity circle. What has gone wrong here is that all the members of this similarity circle are pairwise related, but there is no quality in common to all of them. Goodman (1953, p. 164) calls this problem the "difficulty of imperfect community."

In a large number of cases, analysis based on similarity circles simply fails to return the right classes. Thus, it is fatally flawed as a method of analysis. As intimated earlier, Carnap himself noted the companionship difficulty (§70). He called cases of companion qualities "unfavorable circumstances" and claimed the method of similarity circle analysis works when there are no "systematic connections of qualities." Presumably, Carnap would be tempted to call the imperfect community problem another unfavorable circumstance.

Goodman is skeptical about this invocation of "unfavorable circumstances." He notes that "unfavorable" need not mean "unlikely." After all, given the nature of color vision, might not there be systematic connections between two very slightly different shades of blue (Goodman 1953, p. 161)? (Consider a shade of blue that most of us see only in the fading evening sky and, hence, in momentary experience containing other shades very close to this one. This seems a likely candidate for companionship. Or, again, a rare shade of green that I have seen only in my lover's eyes would presumably be a companion of black for me, since her pupils are black.) More importantly, there is a grave risk that invoking "unfavorable circumstances" is simply circular (Goodman 1953, pp. 162, 164), amounting to nothing more than saying that the method works except in those cases where it fails.

These problems with analysis are problems that Goodman sees with quasi analysis also. For example, Goodman writes (1953, p. 161): "Although this section [§70] describes only a process to which quasianalysis is analogous, the analogy is so close that we have here the essence of Carnap's method of dealing with the problem of abstraction." But, if we recall the differences between analysis and quasi analysis, the situation seems more complicated. The crucial point is that in quasi analysis there is no independent way to characterize the classes the procedure is meant to yield. This is precisely the difference between quasi analysis and analysis. In analysis we assume we have individuals possessed of properties and ask for a way of recovering those properties from a relation description. In quasi analysis, all we have is the relation description and we seek to construct classes that will count as quasi qualities of these pointlike individuals.

Consider, for example, our last world, the one exhibiting the imperfect community problem. To turn this into a quasi analysis problem, we would have to remove the initial listing of qualities of the individuals and present only the relevant relation description. Thus we would start only with the following pair list:

⟨1,1⟩; ⟨1,2⟩; ⟨1,3⟩; ⟨1,5⟩; ⟨1,6⟩
⟨2,1⟩; ⟨2,2⟩; ⟨2,3⟩; ⟨2,5⟩; ⟨2,6⟩
⟨3,1⟩; ⟨3,2⟩; ⟨3,3⟩; ⟨3,4⟩; ⟨3,5⟩; ⟨3,6⟩
⟨4,3⟩; ⟨4,4⟩
⟨5,1⟩; ⟨5,2⟩; ⟨5,3⟩; ⟨5,4⟩; ⟨5,5⟩; ⟨5,6⟩
⟨6,1⟩; ⟨6,2⟩; ⟨6,3⟩; ⟨6,4⟩; ⟨6,5⟩; ⟨6,6⟩

There is no sense in which this pair list "ought to" yield the class {1,2,3,6} as a similarity circle. Indeed, this pair list is identical to the one we could read off from the following world:

1. g	2. g	3. gr
4. r	5. gr	6. gr

In this case, the right classes for analysis to yield would be {1,2,3,5,6} and {3,4,5,6}, which is what the method of similarity circles does yield.

The proper question for quasi analysis is not, Does it yield the right classes? There is no telling what the right classes are, independently of the results of the quasi-analytic procedure itself. What, then, is the relation between analysis and quasi analysis, and what role does discussion of unfavorable circumstances play for Carnap?

The discussion of analysis is motivational and heuristic. Carnap presents a method for constructing classes from relation descriptions that captures an intuitive idea of what a quality is and that works, in a number of cases, to recover the right classes of elements, that is, the classes of objects that do in fact share a property. It is true that sometimes it does not work – sometimes the color kinship relation does not exhibit the right structure to allow the quality classes to be recovered. But in such cases, maybe another relation would help, or perhaps qualities are not distributed in a way that can be recovered by any technique. Nevertheless, a method of some power for recovering properties has been discovered.

But the constitutional problem solved by quasi analysis is quite different. It is not a matter of *recovering* properties from relations; rather it is a matter of defining classes that play the role of properties, given only the structure of a relation. Carnap claims that the method of quasi analysis is useful for this. Indeed, it is more than just useful; Carnap claims not that this method works "in unfavorable circumstances" but that it will always work. In discussing quasi analysis, Carnap (§81) says, for example,

> A more exact investigation, for which there is here no room, teaches, however, that these interferences in concept formation through quasi analysis [e.g., the unfavorable circumstances yielding a companionship problem] only appear if circumstances are present, under which the actual process of cognition, that is, the quasi analysis intuitively carried out in actual life, does not lead to normal results.

What is Carnap saying here? Again, we can see the point by looking at our latest examples: the world exhibiting imperfect community, and

the alternative two-quality world that yields the same pair list. Carnap is not saying that quasi analysis on the pair list yields the wrong results in cases of imperfect community. Rather, he is saying that if the pair list for color kinship is the one we drew up, then quasi analysis yields the two classes it does *and* any agent who had that pair list would conceptualize the world in accordance with the two- and not the three-quality way. For such a person, there would be two qualities arranged in the way shown. This is, I take it, an expression of Carnap's methodological commitment to Gestalt psychology; the Gestalt of the color kinship relation comes first, and qualities are derived from it alone – both in the actual process of knowledge and in quasi analysis. In this sense, in cases of quasi analysis, sensitivity to unfavorable circumstances is a virtue that allows it accurately to reflect the dependence of conceptualization on the empirically given structure of experience.

The notion of "normal results" warrants comment also. For, if there is a normal result, there is some external perspective from which to make this judgment. But this perspective is again empirical psychology. Psychology tells us which color qualities are normal for humans to perceive. The working posit is that the color kinship relation for most of us will then exhibit the structure that allows just these qualities to be constructed. We do not have a pair list of this relation handy, of course, so this does not amount to an empirically testable claim. But then the claim that the reconstruction will yield precisely the results of intuitive concept formation done in our cognitive life is not a straightforwardly empirical claim, and so we ought not to be surprised if it is untestable.

We can treat Goodman's further objections to the more complicated case of analysis and quasi analysis on the basis of part similarity relations in a similar fashion. Goodman (1953, p. 176) considers the offered way of dealing with the problem of the Swedish flags as another case of barring unfavorable circumstances. Suppose, for example, the world contained nothing either yellow or blue but Swedish flags. Then there would be identity between the yellow and blue similarity circles and thus no way of constructing a distinction between (this) blue and (this) yellow. Indeed, the way Carnap distinguishes accidental and essential overlap basically broadens the companionship problem. Goodman (1953, p. 177) identifies the condition that must hold for the process to yield the right results: "We must now assume that fewer than half the [individuals] having any given quality *q* also have one or another of any group of mutually similar qualities other than *q*." This is a very strong assumption. Consider that presumed shade of blue I have seen only in the fading light of the evening sky. I can construct the quality class of that shade of blue only if fewer than half of those experiences have been accompanied by a visceral feeling of well-being (since we must divide visual and emotional qualities only higher up the system). Conceptual wealth is, it seems, purchased at a price.

Needless to say, I think Carnap's response here would be the same as in the earlier cases of "unfavorable circumstances." His methodology commits him to the priority of relational structure. Thus, I see no reason why he should deny the psychological conjecture that someone whose experiential life showed a particularly odd structure would arrive at very skewed concepts, that is, would ascribe properties at variance to the properties most people would ascribe to objects. Quasi analysis gets high marks for respecting this putative cognitive fact.

Goodman's particular concerns dissolve when we respect the constitutional difference between analysis and quasi analysis. In cases of quasi analysis, there is no external perspective against which to check the constructions for correctness; quasi analysis is not constrained by antecedent or independent matters of fact about the qualities of the objects related by the similarity relations. Indeed, to think it was so constrained would be to obviate the need for quasi analysis at the outset and to dismiss Carnap's methodological priority of relations over primitive properties for the basis elements. Goodman's objections do, of course, underscore the need for an account of the standards of constitutional adequacy, the status of notions such as 'unfavorable circumstances,' and the possibility of an external perspective constraining the constitutional system. These topics will concern us throughout the remainder of this work.

CHAPTER THREE

An outline of the constitutional projects for objectivity

We are now in a position to undertake a closer examination of the details of the actual projects for achieving objectivity presented in *Der logische Aufbau der Welt.* An outline of Carnap's levels of constitution is, therefore, in order. In this way, we will be able to see what Carnap is doing in his two projects. Again, many questions about constitution and the order of the definitions will not be addressed in order to focus our attention solely on the question of the way the constitutional system explicates the objectivity of scientific claims.

THE LOWEST LEVELS OF THE CONSTITUTIONAL SYSTEM

The constitutional system meant to mirror the epistemic primacy relation begins, as we have noted, from a single relation, the "recollection of similarity" relation (*Rs*), over total cross-sections of experience at a time. Carnap's first order of business in the constitutional system is to constitute the rich texture of individual psychological life, the autopsychological domain, from this slender basis (cf., esp. §§108–20). Our primary concern is with the role of the autopsychological domain in the construction of objectivity, so I shall only sketch the definitions here. Of primary importance for our purposes is the final accounting of the autopsychological domain and the transition to the world of physics.[1]

Before discussing the definitions, there is a noteworthy aspect of

1 Greater detail on the definitions at the lowest levels of the system can be found in Goodman (1953, chap. 5). Throughout my discussion of the constructions, I have used the abbreviations for Carnap's key terms that George used in his translation, most of which were taken over from Goodman's work. I have done this to facilitate cross-referencing my discussion with the original text and also with Goodman's more detailed discussion.